Section-I: General Ability

Q. No. 1 - 5 Carry One Mark Each

1. Two cars start at the same time from the same location and go in the same direction. The speed of the first car is 50 km/h and the speed of the second car is 60 km/h. The number of hours it takes for the distance between the two cars to be 20 km is _______.

(A) 1  (B) 3  (C) 2  (D) 6

Key:  (C)

Sol:  Let us take two cars A and B

\[ V_A = 50 \text{ km/hr} \]
\[ V_B = 60 \text{ km/hr} \]

Let \( d \) be the distance in km and \( t \) be the time taken in hours.

Time taken = \( \frac{d}{velocity} \)

\[ t = \frac{d}{60} \]  \( \ldots (i) \)
\[ t = \frac{d-20}{50} \]  \( \ldots (ii) \)

Equating (i) and (ii),

\[ \frac{d}{60} = \frac{d-20}{50} \]
\[ d = 60 \]
\[ d = 120 \text{ km} \]

Time taken = \( \frac{120}{60} = 2 \text{ hours} \)

2. The expenditure on the project _______ as follows: equipment Rs.20 lakhs, salaries Rs.12 lakhs, and contingency Rs.3 lakhs.

(A) break  (B) break down  (C) breaks  (D) breaks down

Key:  (D)
3. Ten friends planned to share equally the cost of buying a gift for their teacher. When two of them decided not to contribute, each of the other friends had to pay Rs 150 more. The cost of the gift was Rs _________.

   (A) 12000  (B) 3000  (C) 6000  (D) 666

   **Key:**  (C)

   **Sol:** Let us consider cost of gift = X

   If 10 friends contributes then share = \( \frac{X}{10} \)

   Two friends denied to contribute then remaining are 8 each of remaining to be given 150 more

   \[
   \left( \frac{X}{10} + 150 \right) \times 8 = X
   \]

   \[
   \frac{8X}{10} + 150 \times 8 = X
   \]

   \[
   0.8X + 150 \times 8 = X
   \]

   \[
   0.2X = 1200
   \]

   \[
   X = \frac{1200}{0.2} = 6000 \text{ Rs}
   \]

4. A court is to a judge as ________ is to a teacher.

   (A) a syllabus  (B) a student  (C) a school  (D) a punishment

   **Key:**  (C)

5. The search engine’s business model ________ around the fulcrum of trust.

   (A) sinks  (b) bursts  (C) revolves  (D) plays

   **Key:**  (C)

**Q. No. 6 - 10 Carry Two Marks Each**

6. Three of the five students allocated to a hostel put in special requests to the warden. Given the floor plan of the vacant rooms, select the allocation plan that will accommodate all their requests.

   Request X: Due to pollen allergy, I want to avoid a wing next to the garden.

   Request by Y: I want to live as far from the washrooms as possible, since I am very sensitive to smell.

   Request by Z: I believe in Vaastu and so want to stay in the South-west wing.

   The shaded rooms are already occupied. WR is washroom.
7. The police arrested four criminals – P, Q, R and S. The criminals knew each other. They made the following statements:

P says “Q committed the crime.”
Q says “S committed the crime.”
R says “I did not do it.”
S says “What Q said about me is false.”

Assume only one of the arrested four committed the crime and only one of the statement made above is true. Who committed the crime?

(A) Q  (B) R  (C) S  (D) P

Key: (B)

8. “A recent High Court Judgment has sought to dispel the ideal of begging as a disease which leads to its stigmatization and criminalization – and to regard it as a symptom. The underlying disease is the failure of the state to protect citizens who fall through the social security net.”

Which one of the following statements can be inferred from the given passage?
(A) Begging has to be banned because it adversely affects the welfare of the state
(B) Begging is an offence that has to be dealt with family
(C) Beggars are created because of the lack of social welfare schemes
(D) Beggars are lazy people who beg because they are unwilling to work

Key: (C)

9. In a college, there are three student clubs, Sixty students are only in the Drama club, 80 students are only in the Dance club, 30 students are only in the Maths club, 40 students are in both Drama and Dance clubs, 12 students are in both Dance and Maths clubs, 7 students are in both Drama and Maths clubs, and 2 students are in all the clubs. If 75% of the students in the college are not in any of these clubs, then the total number of students in the college is ________.

(A) 975 (B) 1000 (C) 225 (D) 900

Key: (D)

Sol:

Total number of students in the all three clubs = 60 + 38 + 80 + 5 + 2 + 10 + 30 = 225
Total number of students in the college = X
Given that 75% of students are not of any of these clubs remaining 25% of X = 225
0.25X = 225
X = \frac{225}{0.25} = 900
10. In the given diagram, teachers are represented in the triangle, researchers in the circle and administrators in the rectangle. Out of the total number of the people, the percentage of administrators shall be in the rage of _________.

(A) 16 to 30 (B) 46 to 60 (C) 31 to 45 (D) 0 to 15

Key: (C)

Sol: Teachers

\[ \text{Total number of people} = 70 + 10 + 20 + 20 + 40 = 160 \]
\[ \text{Total number of administrators} = 10 + 20 + 20 = 50 \]
\[ \% \text{ of administrators} = \frac{50}{160} \times 100 = 31.25\% \]

Section-II: Chemical Engineering

Q. No. 1 to 25 Carry One Mark Each

1. Consider a cylinder (diameter D and length D), a sphere (diameter D) and a cube (side length D). Which of the following statements concerning the sphericity \( \Phi \) of the above objects is true:

(A) \( \Phi_{\text{sphere}} > \Phi_{\text{cylinder}} > \Phi_{\text{cube}} \) (B) \( \Phi_{\text{sphere}} = \Phi_{\text{cylinder}} = \Phi_{\text{cube}} \)

(C) \( \Phi_{\text{sphere}} < \Phi_{\text{cylinder}} < \Phi_{\text{cube}} \) (D) \( \Phi_{\text{cylinder}} > \Phi_{\text{sphere}} = \Phi_{\text{cube}} \)
Key:  (A)

Sol:  \( \phi_{\text{sphere}} = 1 \)

\[ \phi_{\text{cube}} = \frac{6/dp}{Sp/Vp} \]

\[ dp = 3 \sqrt{\frac{6V}{\pi}} = dx \sqrt{\frac{6}{\pi}} \]

\[ \phi_{\text{cube}} = \frac{1}{3} \sqrt{\frac{6}{\pi}} \]

\[ \phi_{\text{cylinder}} = \frac{6/dp}{Sp/Vp} \]

\[ dp = \sqrt{\frac{3}{2}} \]

\[ \phi = \left( \frac{3}{\sqrt{2}} \right)^{-1} \]

\( \phi_{\text{sphere}} > \phi_{\text{cylinder}} > \phi_{\text{cube}} \)

2. A system of \( n \) homogeneous linear equations containing \( n \) unknowns will have non-trivial solutions if and only if the determinant of the coefficient matrix is

(A) 1  (B) -2  (C) 0  (D) \( \infty \)

Key:  (C)

Sol: A system of \( n \) linear equations in \( n \) unknowns will have non-trivial solutions \( \iff \det(A) = 0 \).

3. Consider the two countercurrent heat exchanger designs for heating a cold stream from \( t_{in} \) to \( t_{out} \) as shown in figure. The hot process stream is available at \( T_{in} \). The inlet stream conditions and overall heat transfer coefficients are identical in both the designs. The heat transfer area in Design I and Design II are respectively \( A'_{\text{HX}} \) and \( A''_{\text{HX}} \).
If heat losses are neglected, and if both the designs are feasible, which of the following statements holds true:

(A) \( A_{HX}^I > A_{HX}^II \) \( \Rightarrow T_{out}^I < T_{out}^II \)  
(B) \( A_{HX}^I = A_{HX}^II \) \( \Rightarrow T_{out}^I = T_{out}^II \)  
(C) \( A_{HX}^I < A_{HX}^II \) \( \Rightarrow T_{out}^I > T_{out}^II \)  
(D) \( A_{HX}^I < A_{HX}^II \) \( \Rightarrow T_{out}^I = T_{out}^II \)

Key: (B)

Sol: In both figure, counter current process

We know that \( Q = UA\Delta T_{LMTD} \).

If \( A^I = A^II \), \( \Delta T_{LMTD}^I = \Delta T_{LMTD}^II \)

4. For a single component system, vapor (subscript g) and liquid (subscript f) coexist in mechanical, thermal and phase equilibrium when

(A) \( u_g = u_f \) (equality of specific internal energy)  
(B) \( h_g = h_f \) (equality of specific enthalpy)  
(C) \( s_g = s_f \) (equality of specific entropy)  
(D) \( g_g = g_f \) (equality of specific Gibbs free energy)

Key: (D)

Sol: For thermodynamic equilibrium

\( \Delta G = 0, So, Gg = gf \)

5. Three distillation schemes for separating an equimolar, constant relative volatility ABC mixture into nearly pure components are shown. The usual simplifying assumptions such as constant molal overflow, negligible heat loss, ideal trays are valid. All the schemes are designed for minimum total reboiler duty. Given that the relative volatilities are in the ratio \( \alpha_A : \alpha_B : \alpha_C = 8:2:1 \), the correct option that arranges the optimally-designed schemes in ascending order of total reboiler duty is

(A) I, II, III  
(B) III, I, II  
(C) II, I, III  
(D) III, II, I
Key:  (C)

Sol:  For minimum total reboiler duty minimum reflux ratio infinite plates.
So, for $\alpha_A : \alpha_B : \alpha_C = 8 : 2 : 1$

For ascending order of total reboiler duty.
I.

Reboiler duty is being utilized in first column is less but in II column is more $\alpha_B : \alpha_C = 1 : 2$

II.

Minimum reboiler duty in both.

III. Three column so reboiler duty is very high.

6. Consider a rigid, perfectly insulated, container partitioned into two unequal parts by a thin membrane (see figure). One part contains one mole of an ideal gas at pressure $P_i$ and temperature $T_i$ while the other part is evacuated. The membrane ruptures, the gas fills the entire volume and the equilibrium pressure is $P_e = P_i / 4$. If $C_p$ (molar specific heat capacity at constant pressure), $C_v$ (molar specific heat capacity at constant volume) and $R$ (universal gas constant) have the same units as molar entropy, the change in molar entropy $(S_f - S_i)$ is
(A) \( C_p \ln 2 + R \ln 4 \)  
(B) \( -C_r \ln 2 + R \ln 4 \)  
(C) \( R \ln 4 \)  
(D) \( C_p \ln 2 \)

**Key:** (C)  
**Sol:** Insulator container  
No heat transfer  
\[ \Delta T = 0 \text{ for vacuum expansion} \]  
\[ \frac{P_1}{P_2} = \frac{V_2}{V_1} = 4 \]  
For no temperature change  
\[ \Delta S = RLn \frac{V_2}{V_1} = RLn 4 \]

7. The most common catalyst used for oxidation of o-xylene to phthalic anhydride is  
(A) \( V_2O_5 \)  
(B) Pd  
(C) Pt  
(D) Ag  

**Key:** (A)  
**Sol:** Catalyst used for oxidation of 0-xylene to phthalic anhydride is \( V_2O_5 \)

8. For a binary nonideal A-B mixture exhibiting a minimum boiling azeotrope, the activity coefficients, \( \gamma_i \), \( i = A, B \), must satisfy  
(A) \( \gamma_A > 1, \gamma_B > 1 \)  
(B) \( \gamma_A < 1, \gamma_B > 1 \)  
(C) \( \gamma_A = 1, \gamma_B = 1 \)  
(D) \( \gamma_A < 1, \gamma_B < 1 \)  

**Key:** (A)  
**Sol:** For minimum boiling azeotrope  
Total pressure will get increase.  
So, \( y_A p_1^A + y_B x_2 p_2^A > x_1 p_1^A + x_2 p_2^A \)  
For this to be true  
\( y_A & y_B > 1 \)

9. The correct expression for the Colburn j-factor for mass transfer that relates Sherwood number (Sh), Reynolds number (Re) and Schmidt number (Sc) is  
(A) \( \frac{Sh}{(Re)(Sc)^{1/3}} \)  
(B) \( \frac{Sh}{(Re)^{1/2}(Sc)} \)  
(C) \( \frac{Sh}{(Re)^{1/2}(Sc)^{1/3}} \)  
(D) \( \frac{Sh}{(Re)(Sc)} \)  

**Key:** (A)  
**Sol:**  
\[ J = \frac{sh}{Re(Sc)^{1/2}} \]
10. The liquid flow rate through an equal percentage control valve, when fully open, is 150 gal/min and the corresponding pressure drop is 50 psi. If the specific gravity of the liquid is 0.8, then the valve coefficient, \( C_v \) in \( \frac{\text{gal}}{\text{min} \sqrt{\text{psi}}} \) is \( \text{______} \) (rounded off to two decimal places).

Key: (18.97)

Sol: Valve coefficient

\[
C_v = \text{Flow} \times \sqrt{\frac{\text{sp.gravity}}{\text{pressure drop}}}
\]

\[
C_v = 150 \times \sqrt{\frac{0.8}{50}} = 18.97
\]

11. Consider a sealed rigid bottle containing \( \text{CO}_2 \) and \( \text{H}_2\text{O} \) at 10 bar and ambient temperature. Assume that the gas phase in the bottle is pure \( \text{CO}_2 \) and follows the ideal gas law. The liquid phase in the bottle contains \( \text{CO}_2 \) dissolved in \( \text{H}_2\text{O} \) and is an ideal solution. The Henry’s constant at the system pressure and temperature is \( H_{\text{CO}_2} = 1000 \text{bar} \). The equilibrium mole fraction of \( \text{CO}_2 \) dissolved in \( \text{H}_2\text{O} \) is \( \text{______} \) (rounded off to three decimal places).

Key: (0.01)

Sol:

\[
P_{\text{CO}_2} = 10 \text{ bar}
\]

\[
P_{\text{CO}_2} = x_i H
\]

\[
H = 1000 \text{ bar}
\]

\[
x_i = 0.01
\]

12. For a hydraulic lift with dimensions shown in figure, assuming \( g = 10 \text{ m/s}^2 \), the maximum diameter \( D_{\text{lift}} \) (in m) that lifts a vehicle of mass 1000 kg using a force of 100 N is \( \text{______} \) (rounded off to two decimal places).

Key: (0.2)
Sol: Pressure on both sides should be the same

\[
P_A = P_B
\]

\[
\frac{100}{\pi d^2} = \frac{mg}{\pi d_1^2}
\]

\[
\frac{100}{\pi d^2} = \frac{1000 \times 100}{\pi \times 2^2}
\]

\[
d = 0.20m
\]

13. The combination that correctly matches the polymer in Group-1 with the polymerization reaction type in Group-2 is

<table>
<thead>
<tr>
<th>Group-1</th>
<th>Group-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. Nylon 6</td>
<td>I. Condensation polymerization</td>
</tr>
<tr>
<td>Q. Polypropylene</td>
<td>II. Ring opening polymerization</td>
</tr>
<tr>
<td>R. Polyester</td>
<td>III. Addition polymerization</td>
</tr>
</tbody>
</table>

(A) P – II, Q – I, R – III  
(B) P – I, Q – III, R – II  
(C) P – III, Q – II, R – I  
(D) P – II, Q – III, R – I

Key: (D)

Sol: Nylon – 6 Ring opening polymerization  
Polypropylene – Addition polymerization  
Polyester – condensation polymerization.

14. In petroleum refining operations, the process used for converting paraffins and naphthenes to aromatics is

(A) alkylation  
(B) catalytic reforming  
(C) hydrocracking  
(D) isomerization

Key: (B)

Sol: Catalytic reforming is used for converting paraffin’s to aromatic.

15. In Kraft process, the essential chemical reagents used in the digester are

(A) caustic soda, mercaptans and ethylene oxide  
(B) caustic soda, sodium sulphide and soda ash  
(C) quick lime, salt cake and dimethyl sulphide  
(D) baking soda, sodium sulphide and mercaptans
Key:  (B)
Sol:  In Kraft process,
     In digester,
     Caustic soda, sodium sulphide and soda ash is used.

16. Prandtl number signifies the ratio of
   (A) \( \frac{\text{Momentum Diffusivity}}{\text{Thermal Diffusivity}} \)
   (B) \( \frac{\text{Mass Diffusivity}}{\text{Thermal Diffusivity}} \)
   (C) \( \frac{\text{Thermal Diffusivity}}{\text{Momentum Diffusivity}} \)
   (D) \( \frac{\text{Thermal Diffusivity}}{\text{Mass Diffusivity}} \)

   Key:  (A)
Sol:  Prandtl number = Momentum diffusivity thermal diffusivity

17. Producer gas is obtained by
   (A) passing air through red hot coke
   (B) thermal cracking of naphtha
   (C) passing steam through red hot coke
   (D) passing air and steam through red hot coke

   Key:  (D)
Sol:  Producer gas is obtained by passing air and steam through red hot coke.

18. In the drying of non-dissolving solids at constant drying conditions, the internal movement of moisture in the solid has a dominant effect on the drying rate during
   (A) the initial adjustment period only
   (B) the constant rate period only
   (C) the falling rate period only
   (D) both the initial adjustment and constant rate periods

   Key:  (C)
Sol:  The falling rate period only.

19. For a fully-developed turbulent hydrodynamic boundary layer for flow past a flat plate, the thickness of the boundary layer increases with distance \( x \) from the leading edge of the plate, along the free-stream flow direction, as
   (A) \( x^{0.5} \)  (B) \( x^{1.5} \)  (C) \( x^{0.4} \)  (D) \( x^{0.8} \)
Key:  (D)
Sol:  Turbulent flow
\[ \frac{\delta}{x} \propto \frac{1}{\text{Re} y_z} \]
\[ \frac{\delta}{x} \propto \frac{1}{x^{0.2}} \]
\[ \delta \propto x^{0.8} \]

20. Pool boiling equipment operating above ambient temperature is usually designed to operate
(A) far above the critical heat flux
(B) near the critical heat flux
(C) far above the Leidenfrost point
(D) near the Leidenfrost point

Key:  (D)
Sol:  Pool boiling equipment operating near leidenfrost point.

21. For a first order reaction in a porous spherical catalyst pellet, diffusional effects are most likely to lower the observed rate of reaction for
(A) slow reaction in a pellet of small diameter
(B) slow reaction in a pellet of large diameter
(C) fast reaction in a pellet of small diameter
(D) fast reaction in a pellet of large diameter

Key:  (D)
Sol:  For diffusional controlling Fast reaction with large diameter.

22. The desired liquid-phase reaction
\[ D + E \xrightarrow{k_i} F \quad r_f = k_i C_D \times C_E^{0.3} \]
is accompanied by an undesired side reaction
\[ D + E \xrightarrow{k_i} F \quad r_g = k_2 C_D^{0.4} C_E^{1.5} \]
Four isothermal reactor schemes (CSTR: ideal Continuous-Stirred Tank Reactor; PFR: ideal Plug Flow Reactor) for processing equal molar feed rates of D and E are shown in figure. Each scheme is designed for the same conversion. The scheme that gives the most favorable product distribution is:
23. The product of the eigen values of the matrix \[ \begin{pmatrix} 2 & 3 \\ 0 & 7 \end{pmatrix} \] is ____________ (rounded off to one decimal place).

Key: (14)

Sol: ∴ Product of the Eigen values of the matrix \( |A| \)

\Rightarrow Product of the Eigen values = [14 – 0] = 14; Since \( A = \begin{pmatrix} 2 & 3 \\ 0 & 7 \end{pmatrix} \)

24. A thermocouple senses temperature based on the

(A) Nernst Effect
(B) Maxwell Effect
(C) Seebeck Effect
(D) Peltier Effect

Key: (C)

Sol: A thermo couple senses temperature based on seebeck effect.
25. The values of the expression \( \lim_{x \to \frac{\pi}{2}} \frac{\tan x}{x} \) is

\[ \begin{array}{cccc} (A) & \infty & (B) & 0 \\ (C) & 1 & (D) & -1 \end{array} \]

Key: (A)

Sol: \[ \lim_{x \to \frac{\pi}{2}} \frac{\tan x}{x} = \lim_{x \to \frac{\pi}{2}} \frac{\tan(\frac{\pi}{2})}{\frac{\pi}{2}} = \infty \]

26. Carbon monoxide (CO) reacts with hydrogen sulphide \((H_2S)\) at a constant temperature of 800 K and a constant pressure of 2 bar as:

\[ \text{CO} + H_2S \rightleftharpoons \text{COS} + H_2 \]

The Gibbs free energy of the reaction \( \Delta g^\circ_{\text{rxn}} = 22972.3 \text{ J/mol} \) and universal gas constant \( R = 8.314 \text{ J/(mol K)} \). Both the reactants and products can be assumed to be ideal gases. If initially only 4 mol of \( H_2S \) and 1 mol of CO are present, the extent of the reaction (in mol) at equilibrium is________ (rounded off to two decimal places).

Key: (0.29)

Sol: \( T = 800 \text{ K}, P = 2 \text{ bar} \)

\[ \text{CO} + H_2S \rightleftharpoons \text{COS} + H_2 \]

\( \Delta G^\circ = 22972.3 \text{ T/mol} \)

\( R = 8.314 \text{ J/mol K} \)

\( \ln K = \frac{-\Delta G^\circ}{RT} = \frac{-22972.3}{8.314 \times 800} = 0.03162 \)

For ideal gas mix, standard state to be J bar

\( K = K_a = K_p = 0.03162 \)

\( K_p = K_y P^\circ \)

\( Y_{\text{CO}} = \frac{1-\xi}{5}, Y_{\text{H}_2} = \frac{\xi}{5}, Y_{\text{COS}} = \frac{\xi}{5} \)

\( Y_{H_2S} = \frac{4-\xi}{5} \) (\( \because V = 0 \))

Putting the values,

\[ \frac{\xi}{5} \times \frac{\xi}{5} = (0.03162) \times (2)^\circ \]

\[ \xi = 0.2889 \]
27. Consider two non-interacting tanks-in-series as shown in figure. Water enters TANK 1 and $q \text{ cm}^3 / \text{s}$ and drains down to TANK 2 by gravity at a rate $k\sqrt{h_1} \left( \text{cm}^3 / \text{s} \right)$. Similarly, water drains from TANK 2 by gravity at a rate of $k\sqrt{h_2} \left( \text{cm}^3 / \text{s} \right)$ where $h_1$ and $h_2$ represent levels of TANK 1 and TANK 2, respectively (see figure). Drain valve constant $k = 4 \text{ cm}^{2.5} / \text{s}$ and cross-sectional areas of the two tanks are $A_1 = A_2 = 28 \text{ cm}^2$

\[
\begin{align*}
\text{At steady state operation, the water inlet flow rate is } \dot{q}_{\text{ss}} &= 16 \text{ cm}^3 / \text{s}. \\
\text{The transfer function relating the deviation variables } \tilde{h}_2 \left( \text{cm} \right) \text{ to flow rate } \tilde{q} \left( \text{cm}^3 / \text{s} \right) \text{ is} \\
\text{(A) } \frac{2}{\left( 56s + 1 \right)^2} &\quad \text{(B) } \frac{2}{\left( 62s + 1 \right)^2} \quad \text{(C) } \frac{2}{\left( 36s + 1 \right)^2} \quad \text{(D) } \frac{2}{\left( 49s + 1 \right)^2}
\end{align*}
\]

**Key:** (A)

**Sol:** For Non-interacting sys:

\[
\begin{align*}
\text{Fi} &= q(t) \\
\text{Fi} &= K\sqrt{h_1} \\
\text{Fi} &= K\sqrt{h_2}
\end{align*}
\]
Given data

\[ K = 4 \frac{Cm^{2.5}}{s} \]

\[ A_1 = A_2 = 28 \text{ cm}^2 \]

\[ F_{ss} = q_{ns} = 16 \frac{cm^3}{s} \]

At steady state:

\[ F_{ss} = F_{oss} = 16 \frac{cm^3}{s} \]

\[ F_{ss} = F_{oss} = K \sqrt{h_{1ss}} = K \sqrt{h_{2ss}} \Rightarrow 16 = K \sqrt{h_{1ss}} \]

\( \therefore K = 4 \therefore h_{1ss} = 16 \text{cm} = h_{2ss} \)

\[ R_1 = \frac{1}{F_1'(h_{1ss})} = \frac{2 \sqrt{h_{1ss}}}{K} = \frac{2 \sqrt{16}}{4} = 2 \]

\[ R_2 = \frac{1}{F_2'(h_{2ss})} = \frac{2 \sqrt{h_{2ss}}}{K} = \frac{2 \sqrt{16}}{4} = 2 \]

\[ \frac{\bar{h}_2(S)}{\bar{F}_1(S)} = \frac{F_2(S)}{F_1(S)} \cdot \frac{F_1(S)}{F_2(S)} = \frac{1}{(A_1 R_1 S + 1)} \cdot \frac{2}{(A_2 R_2 S + 1)} \]

\[ A_1 = A_2 = 28 \text{ cm}^2 \]

\[ R_1 = R_2 = 2 \]

\[ \frac{\bar{h}_2(S)}{\bar{F}_1(S)} = \frac{2}{(56S+1)} \]

28. A binary mixture with components A and B is to be separated in a distillation column to obtain 95 mol% A as the top product. The binary mixture has a constant relative volatility \( \alpha_{AB} = 2 \). The column feed is a saturated liquid containing 50 mol% A. Under the usual simplifying assumptions such as constant molal overflow, negligible heat loss, ideal trays, the minimum reflux ratio for this separation is____________________(rounded off to one decimal place).

**Key:** (1.7)

**Sol:** Data given

\[ X_p = 0.95 \]

\[ x_f = 0.5 \]

\[ \alpha_{AB} = 2 \]

\[ y = \frac{\alpha x}{1 + (\alpha - 1)x} \]

at \( x = x_f \)
29. The combination that correctly matches the process in Group-1 with the entries in Group-2 is

<table>
<thead>
<tr>
<th>Group-1</th>
<th>Group-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.</td>
<td>Wulff process</td>
</tr>
<tr>
<td>Q.</td>
<td>Sulfite process</td>
</tr>
<tr>
<td>R.</td>
<td>Solvay process</td>
</tr>
<tr>
<td>S.</td>
<td>Frasch process</td>
</tr>
<tr>
<td>I.</td>
<td>Sulfur mining</td>
</tr>
<tr>
<td>II.</td>
<td>Soda ash production</td>
</tr>
<tr>
<td>III.</td>
<td>Acetylene production</td>
</tr>
<tr>
<td>IV.</td>
<td>Pulp production</td>
</tr>
</tbody>
</table>


Key: (B)
Sol: Wulff process for Acetylene production
Sulphite process for pulp production
Solvay process for soda ash production
Frasch process for Sulfur mining

30. Choose the option that correctly matches the step response curves on the left with the appropriate transfer functions on the right. The step input change occurs at time \( t = 0 \).

<table>
<thead>
<tr>
<th>Step response</th>
<th>Transfer function</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. [ \begin{array}{c} \text{y} \ \text{0} \end{array} ] [ \begin{array}{c} \text{t} \end{array} ]</td>
<td>I. [ \frac{K(\xi s + 1)}{s(\tau s + 1)} ] ( K &gt; 0, \xi &gt; 0, \tau &gt; 0 )</td>
</tr>
<tr>
<td>Q. [ \begin{array}{c} \text{y} \ \text{0} \end{array} ] [ \begin{array}{c} \text{t} \end{array} ]</td>
<td>II. [ \frac{K(\xi s + 1)}{s(\tau s + 1)} ] ( K &gt; 0, \xi &gt; \tau &gt; 0 )</td>
</tr>
</tbody>
</table>
Key: (B)
Sol: Step Response

(P) $\frac{K(\xi s + 1)}{S(Zs + 1)}$, $K > 0$
\[ \xi > 0 \]
\[ Z > 0 \]

(Q) $\frac{K(\xi s + 1)}{Zs + 1}$, $K > 0$
\[ \xi > Z > 0 \]

(R) $\frac{K}{(Zs + 1)}$, $K > 0$
\[ Z > 0 \]
31. Consider two competing equipment A and B. For a compound interest rate of 10% per annum, in order for equipment B to be the economically cheaper option, its minimum life (in years) is_______(rounded off to the next higher integer).

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Capital Cost (Rs)</th>
<th>Yearly Operating Cost (Rs)</th>
<th>Equipment Life (Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>80,000</td>
<td>20,000</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>1,60,000</td>
<td>15,000</td>
<td>?</td>
</tr>
</tbody>
</table>

Key: (11)

Sol:

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Capital cost (Rs)</th>
<th>Yearly operating cost (Rs)</th>
<th>Equipment life (Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>80,000</td>
<td>20,000</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>1,60,000</td>
<td>15,000</td>
<td>p</td>
</tr>
</tbody>
</table>

Compound interest rate = 10% per annum

Capitalized cost of A = Capitalized cost B

\[
80.00 + \frac{80,000 - 0}{(1.1)^4 - 1} = 160000 + \frac{160000 - 0}{(1.1)^4 - 1}
\]

\[\therefore\text{ Capitalised cost} = V_0 + \frac{(V_0 - V_f)}{(1+i)^n - 1}\]

Capitalised cost method does not considers operating cost

\[\therefore\text{ operating cost is not used while solving question by solving the question for “n”}\]

\[n = 10.545 \approx 11\text{ years.}\]
32. If \( x, y \) and \( z \) are directions in a Cartesian coordinate system and \( i, j \) and \( k \) are the respective unit vectors, the directional derivative of the function \( u(x, y, z) = x^2 - 3yz \) at the point \((2,0,-4)\) in the direction \((i + j - 2k) / \sqrt{6}\) is \( \text{______________} \) (rounded off to two decimal places).

**Key:** (63.53)

**Sol:**

\[
\text{D.D of } u(x, y, z) = \frac{(i + j - 2k)}{\sqrt{6}} \text{ direction} = \nabla u \frac{(i + j - 2k)}{\sqrt{6}} \quad \ldots (1)
\]

Given \( u(x, y, z) = x^2 - 3yz \)

\[
\Rightarrow \nabla u = \hat{i}(2x) + \hat{j}(-32) + \hat{k}(-3y)
\]

\[
\therefore \text{From (1); D.D of } \frac{u}{P(2,0,-4)} = \frac{2x - 3z + 6y}{\sqrt{6}} / P(2,0,-4)
\]

\[
\Rightarrow \text{D.D of } \frac{u}{P(2,0,-4)} = \frac{4 + 12}{\sqrt{6}} = \frac{16}{\sqrt{6}} = 6.53
\]

33. The elementary liquid-phase irreversible reactions

\[
A \xrightarrow{k_{1}=0.4 \text{ min}^{-1}} B \xrightarrow{k_{2}=0.1 \text{ min}^{-1}} C
\]

take place in an isothermal ideal CSTR (Continuous-Stirred Tank Reactor). Pure \( A \) is fed to the reactor at a concentration of 2 mol/Liter. For the residence time that maximizes the exit concentration of \( B \), the percentage yield of \( B \), defined \( \left( \text{net formation rate of } B \times 100 \right) / \text{consumption rate of } A \) is \( \text{______________} \) (rounded off to the nearest integer).

**Key:** (66.67)

**Sol:**

\[
A \xrightarrow{n=1, k_{1}=0.4 \text{ min}^{-1}} B \xrightarrow{n=1, k_{2}=0.1 \text{ min}^{-1}} C
\]

(Liquid phase irreversible \( R_{xn} \) in CSTR)

Feed is pure \( A \)

\[
C_{A0} = 2 \text{ mol/Liter}
\]

\[
\tau_{\text{optimum}} \text{ for maximum exit concn of } B \text{ in MFR}
\]

\[
\tau_{\text{opt}} = \frac{1}{\sqrt{k_{1}k_{2}}} = \frac{1}{\sqrt{0.4 \times 0.1}} = 5 \text{ min.}
\]
Mole Balance over A:
(Input = out + acc. + disappear – appear)

\[ V_0 C_{A0} = V_0 C_A + 0 + (-V_A V) - 0 \]

\[ V = \frac{C_{A0} - C_A}{(-V_A)} = \frac{C_{A0} - C_A}{K C_A} \]

\[ \tau = \frac{C_{A0} - C_A}{K C_A} \Rightarrow C_A = \frac{C_{A0}}{1 + \tau K_1} \]

\[ C_A = 0.667 \text{ mol}/\mu \]

Mole balance over B:

\[ 0 = \gamma_B C_B + 0 - (K_1 C_A - K_2 C_B) V + 0 \]

(in = out + acc appear + disappear)

\[ \frac{V}{\gamma_0} = \frac{C_B}{r_B - k_1 C_A - k_2 C_B} \]

\[ \tau = \frac{C_B}{k_1 C_A - k_2 C_B} \Rightarrow C_B = 0.889 \text{ mol} \]

Percentage yield of B: \( \frac{\text{Net formation of B (Rate)}}{\text{Net consumption Rate of A}} \times 100 \)

\[ \% Y_{B/A} = \frac{r_B}{(-r_A)} \times 100 = \frac{C_B - C_{B0}}{C_{A0} - C_A} \times 100 (\because C_{B0} = 0) \]

\[ \% Y_{B/A} = \frac{0.889}{2 - 0.667} \times 100 = 66.67\% \]

34. A 20 cm diameter cylindrical solid pellet of a nuclear fuel with density 6000 kg/m³ and conductivity of 300 W/(m K) generates heat by nuclear fission at a spatially uniform rate of \( 10^4 \text{ W/kg} \). The heat from the fuel pellet is transferred to the surrounding coolant by convection such that the pellet wall temperature remains constant at 300 °C. Neglecting the axial and azimuthal dependence, the maximum temperature (in °C) in the pellet at steady state is_________________(rounded off to the nearest integer).

Key: (800)

Sol:

\[ \frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \dot{Q} = 0 \]

\[ \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \dot{Q} r = 0 \]

Integrating: \[ r \frac{dT}{dr} + \frac{\dot{Q} r^2}{2} = C_1 \]
at \( r = 0 \), \( C = 0 \)

\[
\frac{dT}{dr} + \frac{\dot{Q}r}{2} = \quad \text{(i)}
\]

Integrating again:- \( T + \frac{\dot{Q}r^2}{4} = C_2 \)

At \( r = R \), \( T = T_w ; T_w + \frac{\dot{Q}R^2}{4} = C_2 \)

\[
\therefore T + \frac{\dot{Q}r^2}{4} = T_w + \frac{\dot{Q}R^2}{4}
\]

\[
T = T_w + \frac{\dot{Q}}{4}R^2 \left( 1 - \frac{r^2}{R^2} \right) \quad \text{(ii)}
\]

For max. Temp : \( \frac{dT}{dr} = 0 \)

From epb (1) \( r = 0 \) is location for \( T_{\text{max}} \)

\[
T_{\text{max}} = T_w + \frac{\dot{Q}R^2}{4}
\]

\( \dot{Q} = \text{heat generated per unit vol} \)

\[
T_{\text{max}} = 300 \times 10^4 \times 6000 \times \left( 0.1 \right)^2 \times \frac{1}{4 \times 300}
\]

\[
T_{\text{max}} = 800^\circ \text{C}
\]

35. The elementary, irreversible, liquid-phase, parallel reactions \( 2A \rightarrow D \) and \( 2A \rightarrow U \), take place in an isothermal non-ideal reactor. The C-curve measured in a tracer experiment is shown in the figure, where \( C(t) \) is the concentration of the tracer in g/m³ at the reactor exit at time \( t \) (in min).
The rate constants are \( k_1 = 0.2 \text{ Litre (mol min)} \) and \( k_2 = 0.3 \text{ Litre (mol min)} \). Pure A is fed to the reactor at a concentration of 2 mol/Liter. Using the segregated model, the percentage conversion in the reactor is___________(rounded off to the nearest integer).

**Key:** (64.64)

**Sol:**

\[
K_1 = 0.2 \frac{\text{Litre}}{\text{mol min}}
\]

\[
K_2 = 0.3 \frac{\text{Litre}}{\text{mol min}}
\]

\( C(t) \) = tracer concentration

Pure A is fed to reactor.

\( C_{A_6} = 2 \text{ mol/litre} \)

\[
(-r_A) = K_1 C_A^2 + K_2 C_A^2 = (K_1 + K_2) C_A^2 = 0.5 C_A^2 \frac{\text{mol}}{\ell_s}
\]

\% conversion using segregated model:

\[
\bar{x}_A = \int_0^\infty x_A C(t) \, dt \Rightarrow \bar{x}_A = \frac{\int_0^\infty x_A C(t) \, dt}{\int_0^\infty C(t) \, dt}
\]

\[
\therefore E(t) = \frac{C(t)}{\int_0^\infty C(t) \, dt}
\]

\[
\bar{x}_A = \frac{\int_0^1 x_A C(t) \, dt + \int_1^6 x_A C(t) \, dt}{\int_0^6 C(t) \, dt}
\]
for $0 < t < 1$, \( C(t) = 5t \)

for $1 < t < 6$, \( C(t) = (6 - t) \)

For 2\(^{nd}\) order \( R_{sh} \) is constant volume \( R_{shf} \) system:

\[
X_A = \frac{K C_{At}}{1 + K C_{At}} = \frac{0.5 \times 2 \times t}{1 + 0.5 \times 2 \times t} = \frac{t}{1 + t}
\]

\[
X_A = \frac{\int_0^t \frac{1}{1 + t} \times 5t \, dt + \int_t^6 \frac{1}{1 + t} \times (6 - t) \, dt}{15}
\]

\[
X_A = 0.6464 \Rightarrow \% X_A = 64.64\%
\]

36. Consider a vessel containing steam at 180 °C. The initial steam quality is 0.5 and the initial volume of the vessel is 1 m\(^3\). The vessel loses heat at a constant rate \( q \) under isobaric conditions so that the quality of steam reduces to 0.1 after 10 hours. The thermodynamic properties of water at 180 °C are (subscript \( g \): vapor phase; subscript \( f \): liquid phase):

- Specific volume: \( v_g = 0.19405 \text{m}^3/\text{kg} \), \( v_f = 0.001127 \text{m}^3/\text{kg} \)
- Specific internal energy: \( u_g = 2583.7 \text{kJ/kg} \), \( u_f = 762.08 \text{kJ/kg} \)
- Specific enthalpy: \( h_g = 22778.2 \text{kJ/kg} \), \( h_f = 763.21 \text{kJ/kg} \)

The rate of heat loss \( q \) (in kJ/hour) is _________ (rounded off to the nearest integer).

**Key:** (747)

**Sol:**

\[ V = m \left[ x_v \rho_g + (1-x_v) \rho_f \right] \]

\[ 1 = m \left[ 0.5 \times 0.19405 \times 5 \times 0.001127 \right] \]

\[ m = 10.24711 \text{ kg of mix.} \]

5.123555 kg of steam and

5.123555 kg of saturated water

\[ i_L = (u_g - u_f) + 1w_2 \]

\[ = \left[ x_v u_g + (1-x_v) u_f \right] - \left[ x_v u_g + (1-x_v) u_f \right] \]

\[ = \left[ u_g (x_v - x_f) - (x_v - x_f) u_f \right] \]

\[ = (x_v - x_f) (u_g - u_f) \]

\[ = (0.1 - 0.5) (2583.7 - 762.08) \]

\[ = -728.648 \text{ KJ/Kg} \]

\[ \text{Heat loss per} = \frac{728.648 \times 10.24711}{10} \]

\[ = 746.65 \text{ KJ/hr} \]
37. For the closed loop system shown in figure, the phase margin in degrees) is ___________ (rounded off to one decimal place).

Key: (45.375)

Sol:

Open loop transfer function

\[ G_{OLTS} = 5 \times \left( \frac{e^{-0.1s}}{0.5s + 1} \right) \]

Dead time of the system, \( t_d = 0.1 \) Zp of the system = \( Z_p = 0.5 \) (1st order system) phase margin,

\[ PM = 180^\circ - |\phi'| \]

(\( \phi' \) is the value of phase at gain crossover frequency)

At gain crossover frequency, AR = 1

\[ AR \text{ of system} = 5 \times \frac{1}{\left( \frac{1}{AR \text{ of controller}} \times \frac{1}{AR \text{ of Dead time}} \times \sqrt{1 + (\frac{0.5\omega}{\sqrt{AR \text{ of 1st order process}}})^2} \right)} \]

at \( AR = 1 \), \( \omega = 9.7979 \text{ rad/time} \)

\[ \phi' = -t_d \omega \times \frac{180^\circ}{\pi} + \tan^{-1} \left( -\frac{\omega}{\sqrt{0.5\omega}} \right) + \theta \]

Phase angle for Dead time Phase angle for 1st order system Phase angle for P controller

at \( \omega = 9.7979 \text{ rad/time} \)

\[ \phi' = -56.16156^\circ - 78.46297^\circ = -134.6245^\circ \]

PM = \( 180^\circ - |\phi'| = 45.37546^\circ \)
38. A countercurrent absorption tower is designed to remove 95% of component A from an incoming binary gas mixture using pure solvent B. The mole ratio of A in the inlet gas is 0.02. The carrier gas flow rate is 50 kmol/h. The equilibrium relation is given by \( Y = 2X \), where \( Y \) and \( X \) are the mole ratios of A in the gas and liquid phases, respectively. If the tower is operated at twice the minimum solvent flow rate, the mole ratio of A in the exit liquid stream is__________ (rounded off to three decimal places).

**Key:** (0.005)

**Sol:**

\[
\begin{align*}
&G_s = \frac{50 \text{ kmol}}{\text{hr}}, 1/2 \\
&Y_1 = 0.02 \\
&Y = 2X \\
&G_s X_1 + L_s X_2 = G_s Y_2 + L_s X_1 \\
&Y = 2X \text{ given equal relation} \\
&\text{if } Y_1 = 0.02 \\
&\text{then } X_1^* = \frac{0.02}{2} = 0.01 \\
&G_s Y_1 + L_{s,\text{min}} X_2 = G_s Y_2 + L_{s,\text{min}} X_1^* \\
&L_{s,\text{min}} = 95 \text{ Kmol/hr} \\
&L_{s,\text{opt}} = 2 \times L_{s,\text{min}} \\
&\text{Given} \\
&L_{s,\text{opt}} = 190 \text{ Kmol/hr} \\
&\text{By using } L_{s,\text{opt}} \\
&G_s Y_1 + L_{s,\text{opt}} X_2 = G_s Y_2 + L_{s,\text{opt}} X_1 \\
&X_1 = 0.005
\end{align*}
\]
39. A centrifugal pump is used to pump water (density $1000 \text{ kg/m}^3$) from an inlet pressure of $10^5 \text{ Pa}$ to an exit pressure of $2 \times 10^5 \text{ Pa}$. The exit is at an elevation of 10 m above the pump. The average velocity of the fluid is 10 m/s. The cross-sectional area of the pipes at the pump inlet and outlet is $10^{-3} \text{ m}^2$ and acceleration due to gravity is $g = 10 \text{ m/s}^2$.

Neglecting losses in the system, the power (in Watts) delivered by the pump is_________ (rounded off to the nearest integer).

Key: (2)

Sol:

$P_1 = 10^5 \text{ Pa}$

$V_1 = 10 \text{ m/s}$

$A_1 = 10^{-6} \text{ m}$

$P_2 = 2 \times 10^5 \text{ Pa}$

$V_2 = 10 \text{ m/s}$

$A_2 = 10^{-3} \text{ m}^2$

$g = 10 \text{ m/s}^2$

$\rho = 1000 \text{ kg/m}^3$

Applying Bernoulli equation:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + y_1 + H_{pu\text{mp}} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + y_2$$

$H_{pu\text{mp}} = \left(\frac{2 \times 10^5}{\rho g} - \frac{10^5}{\rho g}\right) + 10$

$H_{pu\text{mp}} = \frac{10^5}{1000 \times 10} + 10 = 20 \text{ m}$

Power of pump = $\omega Q H$

$Q = AV = 10^{-3} \times 10 = 10^{-2} \text{ m}^3/\text{s}$

$\omega = \rho g = 1000 \times 10 = 10^4 \text{ kg/m}^2\text{s}^2$

$P = \omega Q H = 10^4 \times 10^{-2} \times 20 = 2000 \text{ W}$

$P = 2 \text{ kW}$
40. The solution of the ordinary differential equation
\[ \frac{dy}{dx} + 3y = 1, \] subject to the initial condition \( y = 1 \) at \( x = 0 \), is

(A) \( \frac{1}{3}(1 + 2e^{-x/3}) \)
(B) \( \frac{1}{3}(5 - 2e^{-x/3}) \)
(C) \( \frac{1}{3}(5 - 2e^{-3x}) \)
(D) \( \frac{1}{3}(1 + 2e^{-3x}) \)

Key: (D)

Sol: Given D.E is \( \frac{dy}{dx} + 3y = 1 \) \( \ldots(1) \)

\( y = 1 \) at \( x = 0 \)

\( P = 3; \, Q = 1 \)

\( I.F = e^{\int P \, dx} = e^{3x} \)

Solution of equation (1), \( y.e^{3x} = \int 1.e^{3x} \, dx \)

\( \Rightarrow ye^{3x} = \frac{e^{3x}}{3} + c \)

\( \Rightarrow y = \frac{1}{3} + ce^{-3x} \) \( \ldots(2) \)

\( \therefore y = 1 \) at \( x = 0 \)

(2) \( \Rightarrow 1 = \frac{1}{3} + C \Rightarrow C = \frac{2}{3} \)

(2) \( \Rightarrow y = \frac{1}{3} + \frac{2}{3}e^{-3x} = \frac{1}{3}(1 + 2e^{-3x}) \)

\( \Rightarrow y = \frac{1}{3}(1 + 2e^{-3x}) \)

41. A disk turbine is used to stir a liquid in a baffled tank. To design the agitator, experiments are performed in a lab-scale model with a turbine diameter of 0.05 m and a turbine impeller speed of 600 rpm. The liquid viscosity is 0.001 Pa s while the liquid density is 1000 kg/m³. The actual application has a turbine diameter of 0.5 m, an impeller speed of 600 rpm, a liquid viscosity of 0.1 Pa s and a liquid density of 1000 kg/m³. The effect of gravity is negligible. If the power required in the lab-scale model is \( P_1 \) and the estimated power for the actual application is \( P_2 \) then the ratio \( P_2 / P_1 \) is

(A) \( 10^3 \) \hspace{1cm} (B) \( 10^4 \) \hspace{1cm} (C) \( 10^5 \) \hspace{1cm} (D) \( 10^6 \)
Key: (C)
Sol: Design parameters of disk turbine in baffled tank are:

<table>
<thead>
<tr>
<th>Lab scale model</th>
<th>Prototype</th>
</tr>
</thead>
<tbody>
<tr>
<td>D, Diameter = 0.05 m</td>
<td>Diameter = 0.5 m</td>
</tr>
<tr>
<td>N, impeller speed = 600 rpm</td>
<td>Impeller speed = 600 rpm</td>
</tr>
<tr>
<td>liq. Viscosity = 0.001 pa. &amp; ρ, liq density=100 kg/m³</td>
<td>Liq. Viscosity = 0.1 pa%</td>
</tr>
<tr>
<td>$N_{Re, mix} = \frac{1000 \times 600 \times (0.05)^2}{0.001}$</td>
<td>Liq. Density = 100 kg/m³</td>
</tr>
<tr>
<td>$N_{Re, mix} = 1500000$</td>
<td>$N_{Re, mix} = 1500000$</td>
</tr>
</tbody>
</table>

For $N_{Re, mix} > 10^4$

Power no. $N_p = \frac{P}{ρN^3D^5} = \text{Constant}$

$$\frac{P_1}{ρ_1N_1^3D_1^5} = \frac{P_2}{ρ_2N_2^3D_2^5} \quad \left\{ \rho_1 = \rho_2 \right\} \quad \left\{ N_1 = N_2 \right\}$$

$$\frac{P_2}{P_1} = \left( \frac{D_2}{D_1} \right)^5 = \left( \frac{0.5}{0.05} \right)^5 = 10^5$$

42. An incompressible Newtonian fluid flows in a pipe of diameter $D_1$ at volumetric flow rate $Q$. Fluid with same properties flows in another pipe of diameter $D_2 = D_1/2$ at the same flow rate $Q$. The transition length required for achieving fully-developed flow is $l_1$ for the tube of diameter $D_1$, while it is $l_2$ for the tube of diameter $D_2$. Assuming steady laminar flow in both cases, the ratio $l_1/l_2$ is:

(A) 1/4  \quad (B) 1  \quad (C) 2  \quad (D) 4

Key: (B)
Sol:
Fully developed laminar flow

\[ Q_1 = Q_2 = Q \]
\[ D_1 = D, \quad D_2 = D/2 \]
\[ \ell \frac{d}{d} \text{ is 0.006} \]
\[ \ell \propto R \]
\[ \ell \propto \frac{\rho V d^2}{\mu} \]
\[ \ell \propto Vd \]

\[ \frac{\ell_1}{\ell_2} = \frac{V_d^1}{V_d^2} = \frac{\pi V_d^1}{4} \]
\[ \frac{\ell_1}{\ell_2} = \frac{Q_1}{Q_2} = 1 \]
\[ \therefore Q_1 = Q_2 \]

43. The elementary irreversible gas-phase reaction \( A \rightarrow B + C \) is carried out adiabatically in an ideal CSTR (Continuous-Stirred Tank Reactor) operating at 10 atm. Pure A enters the CSTR at a flow rate of 10 mol/s and a temperature of 450 K. Assume A, B and C to be ideal gases. The specific heat capacity at constant pressure \( C_p \) and heat of formation \( H_0^i \) of component \( i = A, B, C \) are:

\[ C_{pa} = 30J/(\text{mol K}) \quad C_{pb} = 10J/(\text{mol K}) \quad C_{pc} = 20J/(\text{mol K}) \]
\[ H_0^A = 90kJ/mol \quad H_0^B = -54kJ/mol \quad H_0^C = -45kJ/mol \]

The reaction rate constant \( k(\text{per second}) = 0.133 \exp \left( \frac{E}{R \left( \frac{1}{450} - \frac{1}{T} \right)} \right) \) where \( E = 31.4kJ/mol \) and universal gas constant \( R = 0.082 \text{ L atm/(mol K)} = 8.314 \text{ J/(mol K)} \). The shaft work may be neglected in the analysis, and specific heat capacities do not vary with temperature. All heats of formation are referenced to 273 K. The reactor volume (in Liters) for 75% conversion is_________ (rounded off to the nearest integer).

**Key:** (89)

44. For a given binary system at constant temperature and pressure, the molar volume (in m\(^3\) mol) is given by: \( u = 30x_A + 20x_B + x_A x_B \left( 15x_A - 7x_B \right) \), where \( x_A \) and \( x_B \) are the mole fractions of components A and B, respectively. The volume change of mixing \( \Delta u_{\text{mix}} \) (in m\(^3\) mol) at \( x_A = 0.5 \) is_________ (rounded off to one decimal place).
Key: (1)
Sol: At const temperature and pressure, molar volume of a binary system is given by:
\[ V = 30x_A + 20x_B + x_A x_B (15x_A - 7x_B) \]  
\[ x_A \text{ and } x_B \] are mol fractions of components A and B in sol.

Volume change of mixing
\[ \Delta V = V - I(V_i x_i) \]
\[ \Delta V = \text{Volume change of mixing} \]
\[ V = \text{Molar volume of sol} \]
\[ V_i = \text{Molar volume of component } i \text{ in pure form.} \]
\[ \Delta V = V - (V_1 x_1 + V_2 x_2) \]
As \[ V_i \] = molar volume of component “1” is pure form
\[ V_1 | x_{1,i} = 30 \text{ (m}^3/\text{mol}) \]
\[ V_2 | x_{2,i} = 20 \text{ (m}^3/\text{mol}) \]
\[ \Delta V = V - (V_1 x_1 + V_2 x_2) \]
\[ \Delta V = 30x_1 + 20x_2 + x_1 x_2 (15x_1 - 7x_2) - (30x_1 + 20x_2) \]
\[ \Delta V = x_1 x_2 (15x_1 - 7x_2) \]
\[ \Delta V |_{x_1=0.5, x_2=0.5} = 0.5 \times 0.5 \times [15 \times 0.5 - 7 \times 0.5] \]
\[ \Delta V |_{x_1=0.5, x_2=0.5} = 1 \text{ (m}^3/\text{mol}) \]

45. Stream A with specific heat capacity \[ C_{PA} = 2000 \text{ J/(kg K)} \] is cooled from 90°C to 45°C in a concentric double pipe counter current heat exchanger having a heat transfer area of 8m². The cold stream B of specific heat capacity \[ C_{PB} = 1000 \text{ J/(kg K)} \] enters the exchanger at a flow rate 1 kg/s and 40°C. The overall heat transfer coefficient \[ U = 250 \text{ W/(m}^2\text{K}) \]. Assume that the mean driving force is based on the arithmetic mean temperature difference, that is
\[ [\Delta T]_{AMTD} = \left( \frac{T_{A,in} + T_{A,out}}{2} \right) - \left( \frac{T_{B,in} + T_{B,out}}{2} \right) \]  
where \[ T_{i,in} \] and \[ T_{i,out} \] refer to the temperature of the \( i \)-th stream (\( i = A, B \)) at the inlet and exit, respectively. The mass flow rate of stream A (in kg/s), is________________(rounded off to two decimal places).

Key: (0.25)
Sol:
\[ C_{PA} = C_h = 200 \frac{J}{\text{Kg K}} \]
\[ C_{PB} = C_c = 100 \frac{J}{\text{Kg K}} \]
46. A fractionator recovers 95 mol% n-propane as the distillate from an equimolar mixture of n-propane and n-butane. The condensate is a saturated liquid at 55°C. The Antoine equation is of the form, \[ \ln(p_{sat} \text{ in bar}) = A - \frac{B}{T \text{[in K]} + C} \]; and the constants are provided below:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-propane</td>
<td>9.1058</td>
<td>1872.46</td>
<td>-25.16</td>
</tr>
<tr>
<td>n-butane</td>
<td>9.0580</td>
<td>2154.90</td>
<td>-34.42</td>
</tr>
</tbody>
</table>

Assuming Raoult’s law, the condenser pressure (in bar) is ________ (rounded off to one decimal place)

Key: (17.9)

47. A solid sphere of radius 1 cm and initial temperature of 25°C is exposed to a gas stream at 100°C. For the solid sphere, the density is 10^4 kg/m^3 and the specific heat capacity is 500 J/(kg K). The density of the gas is 0.6 kg/m^3 and its specific heat capacity is 10^3 J/(Kg K). The solid sphere is approximated as a lumped system (Biot number <<1) and all specific heats are constant. If the heat transfer coefficient between the solid and gas is 250 W/(m^2 K), the time (in seconds) needed for the sphere to reach 95°C is ____________ (rounded off to the nearest integer)

Key: (903)
Sol: \( \ell n \left( \frac{T - T_p}{T_p - T_\infty} \right) = -\frac{ht}{\rho LC} \)

\( \ell n \left( \frac{95 - 100}{25 - 100} \right) = -\frac{50t}{10^4 \times \frac{1}{3\times100} \times 500} \)

\(-2.708 = -3 \times 10^{-3} t \)
\(t = 902.68 \text{ seconds}\)

48. Two unbiased dice are thrown. Each dice can show any number between 1 and 6. The probability that the sum of the outcomes of the two dice is divisible by 4 is_________ (rounded off to two decimal places).

Key: (0.25)

Sol: Two dice are rolled.

\( \therefore \) Total number of outcomes, \( n(s) = 6^2 = 36 \)

Let ‘A’ denote the sum of the outcomes of the two dice is divisible by 4 \{4, 8 or 12\}

\( \therefore \) No. of favorable cases, \( n(A) = 3 + 5 + 1 = 9 \)

\( \therefore \) (1,3) (2, 2) (3, 1) | (2, 6) (3, 5) (4, 4) (5, 3) (6, 2) | (6, 6)

\( \therefore \) Required probability \( P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{9}{36} = \frac{1}{4} = 0.25 \)

49. Consider the reactor-separator-recycle process operating under steady state conditions as shown in the figure. The reactor is an ideal Continuous-Stirred Tank Reactor (CSTR), where the reaction \( A + B \rightarrow C \) occurs. Assume that there is no impurity in the product and recycle streams. Other relevant information are provided in the figure. The mole fraction of B \( (x_B) \) in the reactor that minimizes the recycle rate is____________________(rounded off to two decimal places).
Key: (*)

50. The value of the complex number $i^{-\frac{1}{2}}$ (where $i = \sqrt{-1}$) is

(A) $\frac{1}{\sqrt{2}}(1-i)$  
(B) $-\frac{1}{\sqrt{2}}i$  
(C) $\frac{1}{\sqrt{2}}i$  
(D) $\frac{1}{\sqrt{2}}(1+i)$

Key: (A)

Sol: $i^{-\frac{1}{2}} = e^{\left(\frac{-\pi}{4}\right)} = \cos\left(\frac{-\pi}{4}\right) + isin\left(\frac{-\pi}{4}\right) = \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}(1-i)$

51. A taxi-car is bought for Rs. 10 lakhs. Its salvage value is zero. The expected yearly income after paying all expenses and applicable taxes is Rs. 3 lakhs. The compound interest rate is 9% per annum. The discounted payback period (in years), is___________________(rounded off to the next higher integer).

Key: (5)

Sol: Taxi-car bought for Rs 10 Lakhs

∴ Fixed capital investment = 10 Lakhs Rs

Salvage value $V_s = 0$

Profit after all the taxes & expenses = 3 Lakhs Rs

Compound interest rate = 9% per Annam

Discounted payback period =?

$10 \text{ Lakhs} = \frac{3 \text{ Lakhs}}{(1+i)^1} + \frac{3 \text{ Lakhs}}{(1+i)^2} + \frac{3 \text{ Lakhs}}{(1+i)^3} + \ldots + \frac{3 \text{ Lakhs}}{(1+i)^n}$

$P = \frac{R}{i} \left[ \frac{(1+i)^n - 1}{(1+i)^n} \right]$  

$10 \text{ Lakhs} = \frac{3 \text{ Lakhs}}{0.9} \left[ \frac{(1+0.09)^n - 1}{(1+0.09)^n} \right]$  

$0.3 \times (1.09)^n = (1.09)^n - 1$

$1 = (1.09)^1 - 3(1.09)^n = 0.7(1.09)^n$

$\frac{1}{0.7} = (1.09)^n \Rightarrow n\ln(1.09) = \ln\left(\frac{1}{0.7}\right)$

$n = \frac{\ln(1/0.7)}{\ln(1.09)} = 4.1388 \text{ years} \approx 5 \text{ years}$
52. 100 kg of a feed containing 50 wt. % of a solute C is contacted with 80 kg of a solvent containing 0.5 wt. % of C in a mixer-settler unit. From this operation, the resultant extract and raffinate phases contain 40 wt.% and 20 wt.% of C, respectively. If E and R denote the mass of the extract and raffinate phases, respectively, the ratio E/R is

(A) 1/4  (B) 1/2  (C) 2/3  (D) 1

**Key:**  (C)

**Sol:**

\[ F x_f + S y_s = R x_R + E y_E \]

\[ 100 \times 0.5 + 80 \times 0.005 = R \times 0.2 + E \times 0.4 \]

\[ 0.2R + 0.4E = 50 + 0.4 = 50.4 \]  \( \ldots (1) \)

\[ F + S = R + E \]

\[ 100 + 80 = R + E \]

\[ R + E = 180 \]  \( \ldots (2) \)

\[ R = 180 - E \]  \( \ldots (3) \)

Putting equation (3) in equation (1)

\[ 0.2(180 - E) + 0.4E = 50.4 \]

\[ E = 72 \]  \( \ldots (4) \)

Putting equation (4) in equation (3)

\[ R = 180 - E = 180 - 72 = 108 \]

\[ \frac{E}{R} = \frac{72}{108} = \frac{2}{3} \]

53. Two spherical camphor particles of radii 20 cm and 5 cm, far away from each other, are undergoing sublimation in a stream of air. The mass transfer coefficient is proportional to \( 1/\sqrt{r(t)} \), where \( r(t) \) is the radius of the sphere at time \( t \). Assume that the partial pressure of camphor far away from the surface of the particle is zero. Also, assume quasi-steady state, identical ambient conditions, and negligible heat effects. If \( t_1 \) and \( t_2 \) are the times required for complete sublimation of the 20 cm and 5 cm camphor particles, respectively, the ratio \( t_1 / t_2 \) is______________ (rounded off to one decimal place).

**Key:**  (8)

**Sol:**

Radii of two given spherical camphor particles are 20 cm and 5 cm (given) mass transfer coefficient \( \propto \frac{1}{\sqrt{r(t)}} \)

\( r(t) \) radius of sphere at any time “t” partial pressure of camphor for away from surface of particle is zero

\( \therefore \) at quasi –static condition
Flux : $-\frac{N_A}{r} = \frac{PD_{AB}}{RT_{ri}} \frac{(P_{A1} - P_{A2})}{P_{BLM}}$

$N_A = K_G \Delta P$ where $K_G = \frac{PD_{AB}}{RT_{ri}P_{BLM}}$

Time taken for complete evaporation :-

$$t = \frac{\rho_i RTP_{BLM}}{2M_A PD_{AB} \Delta P_A} = \frac{\rho_i}{2M_A} \times \frac{1}{N_A} \text{ when}$$

$t \propto \frac{r_i}{(1/\sqrt{r_i})} \Rightarrow t \propto r_i^{3/2}$

$$\frac{t_1}{t_2} = \left( \frac{r_1}{r_2} \right)^{3/2} = \left( \frac{20}{5} \right)^{3/2} = 8$$

54. A first-order irreversible liquid phase reaction $A \rightarrow B (k = 0.1 \text{ min}^{-1})$ is carried out under isothermal, steady state conditions in the following reactor arrangement comprising an ideal CSTR (Continuous-Stirred Tank Reactor) and two ideal PFRs (Plug Flow Reactors).

From the information in the figure, the volume of the CSTR (in Liters) is ______________ (rounded off to the nearest integer).
A → B \( (K = 0.1 \text{ m}^0 \text{ m}^{-1}) \) at isothermal, steady state conditions

\( (1^{\text{st}} \text{ order } R \times n) \)

PFRS in series is replaced by a single PFR of same equivalent volume.

\[ \therefore V_p = V_{p1} + V_{p2} = 40 + 60 = 100 \ell \]

For PFR

\[ \frac{V_{0.2}}{C_{A0}} = \int \frac{dC_A}{-r_A} = -\int \frac{dC_A}{KC_A} \]

\[ = -\frac{1}{C_{A0}} \ln \left( \frac{C_A}{C_{A0}} \right) = \frac{1}{0.1} \ln \left( \frac{0.5}{0.5} \right) \]

\[ \frac{100}{V_{0.2}} = \frac{1}{0.1} \ln \left( \frac{0.5}{0.5} \right) \Rightarrow V_{0.2} = 14.4269 \ell/\text{min} \]

\[ V_0 = V_{0.1} + V_{0.2} \Rightarrow V_{0.1} = V_0 - V_{0.2} \]

\[ V_{0.1} = 100 - 14.4269 = 85.573 \ell/\text{min} \]

For MFR

\[ \frac{V_m}{C_{A0}} = \frac{C_{A0} - C_A}{-r_A} \]

\[ = \frac{C_{A0} - C_A}{KC_A} = \frac{1 - 0.5}{0.1 \times 0.5} \]

\[ V_m = 85.573 \times \left( \frac{1 - 0.5}{0.1 \times 0.5} \right) = 913.44 \ell/\text{t} \]

55. The Newton-Raphson method is used to determine the root of the equation \( f(x) = e^{-x} - x \).

If the initial guess for the root is 0, the estimate of the root after two iterations is_______ (rounded off to three decimal places).

**Key:** (0.57)

**Sol:** Let \( f(x) = e^{-x} - x \& x_0 = 0 \)

\[ \Rightarrow f'(x) = -e^{-x} - 1 \]

Using Newton-Raphson method, the iterative formula

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

1st iteration: \( \therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \)

\[ \Rightarrow x_1 = 0 - \frac{f(0)}{f'(0)} = -\frac{1}{-1 - 1} = \frac{1}{2} = 0.5 \]

2nd iteration: \( \therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \)

\[ \Rightarrow x_2 = 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.5 - \frac{0.107}{-1.607} \approx 0.57 \]