GENERAL APTITUDE

Q. No. 1 - 5 Carry One Mark Each

1. Five different books (P, Q, R, S, T) are to be arranged on a shelf. The books R and S are to be arranged first and second, respectively from the right side of the shelf. The number of different order in which P, Q and T may be arranged is __________.

(A) 2 (B) 120 (C) 6 (C) 12

Key: (C)

Sol:

$$\underset{\substack{\downarrow \\ 3 \\ \text{choices}}}{\underline{\quad}}\ \times\ \underset{\substack{\downarrow \\ 2 \\ \text{choices}}}{\underline{\quad}}\ \times\ \underset{\substack{\downarrow \\ 1 \\ \text{choice}}}{\underline{\quad}}\quad \underline{S}\quad \underline{R}$$

(P, Q, T)

$\therefore$ The number of different orders in which P, Q and T arranged = $3 \times 2 \times 1 = 6$

2. The boat arrived ________ dawn.

(A) on (B) at (C) under (D) in

Key: (B)

3. It would take one machine 4 hours to complete a production order and another machine 2 hours to complete the same order. If both machines work simultaneously at their respective constant rates, the time taken to complete the same order is __________ hours.

(A) 2/3 (B) 7/3 (C) 4/3 (D) 3/4

Key: (C)

Sol: Machine 1 takes to complete a work = 4 hours

$\Rightarrow$ Machine 1, 1 hour work $= \frac{1}{4}$

Machine 2 takes to complete the same work = 2 hours

$\Rightarrow$ Machine 2, 1 hour work $= \frac{1}{2}$

(Machine 1+ Machine 2) $\to$ 1 hour work $= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

$\because$ Required time $= \frac{\text{Total work}}{\text{1hr work}} = \frac{1}{\left(3/4\right)} = \frac{4}{3}$ hours

4. When he did not come home, she _______ him lying dead on the roadside somewhere

(A) concluded (B) pictured (C) notice (D) looked

Key: (B)

5. The strategies that the company ________ to sell its products ________ house-to-house marketing.

(A) uses, include (B) use, includes (C) uses, including (D) used, includes

Key: (A)

Q. No. 6 - 10 Carry Two Marks Each

6. "Indian history was written by British historians – extremely well documented and researched, but not always impartial. History had to serve its purpose: Everything was made subservient to the glory of the Union Jack. Latter-day Indian scholar presented a contrary picture."

From the text above, we can infer that:

Indian history written by British historians ________.

(A) was well documented and not researched but was always biased

(B) was not well documented and researched and was sometimes biased

(C) was well documented and researched but was sometimes biased

(D) was not well documented and researched and was always biased

Key: (C)

7. Two design consultants, P and Q, started working from 8 AM for a client. The client budgeted a total of USD 3000 for the consultants. P stopped working when the hour hand moved by 210 degrees on the clock. Q stopped working when the hour hand moved by 240 degrees. P took two tea breaks of 15 minutes each during her shift, but took no lunch break. Q took only one lunch break for 20 minutes, but no tea breaks. The market rate for consultants is USD 200 per hour and breaks are not paid. After paying the consultants, the client shall have USD ________ remaining in the budget.

(A) 000.00 (B) 433.33 (C) 166.67 (C) 300.00

Key: (C)

Sol: Given, P and Q started working from 8 A.M for a client

Total budget = USD 3000

P worked exactly 7 hours but took 30 min break.

'P' working number of hours = 6.5 hours

'Q' worked exactly 8 hours but took 20 min break.

'Q' working number of hours ≈ 7.67 hours

Client paying for both P and Q = USD 200/hr

$\therefore$ Total USD Paid = 6.5 × 200 + 7.67 × 200 = 1300 +1534 = 2834

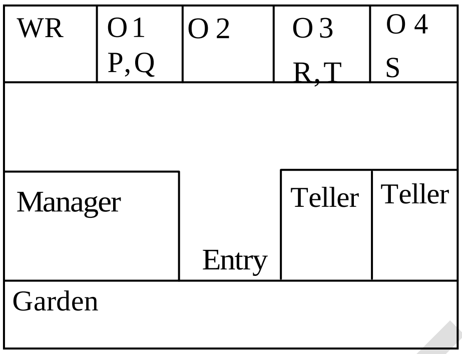
After paying the consultants P and Q, the client shall have USD remaining in the budget

$= 3000 - 2834 \approx 166$

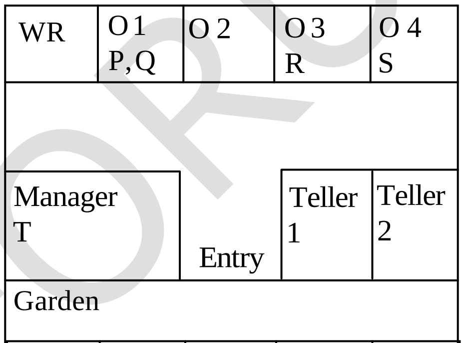
8. Five people P, Q, R, S and T work in a bank . P and Q don’t like each other but have to share an office till T gets a promotion and moves to the big office next to the garden. R, who is currently sharing an office with T wants to move to the adjacent office with S, the handsome new intern. Given the floor plan, what is the current location of Q, R and T?

(O = Office, WR = Washroom)

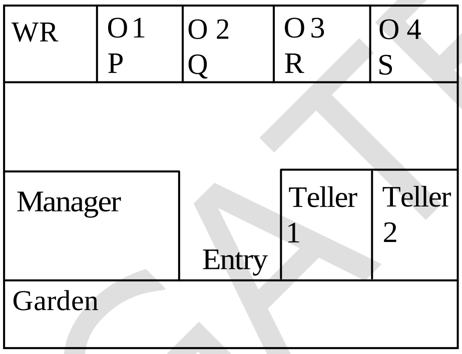
(A)



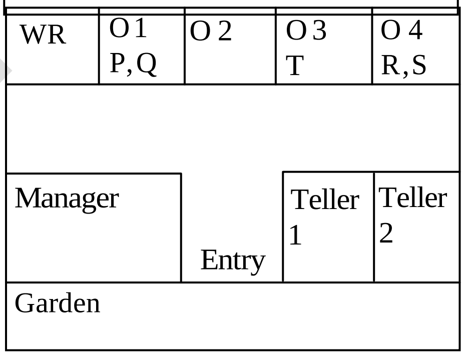
(B)



(C)



(D)



Key: (A)

Sol: According to the given data, Option (A) is correct.

9. Four people are standing in a line facing you. They are Rahul, Mathew, Seema and Lohit. One is an engineering, one is a doctor, one a teacher and another a dancer. You are told that:

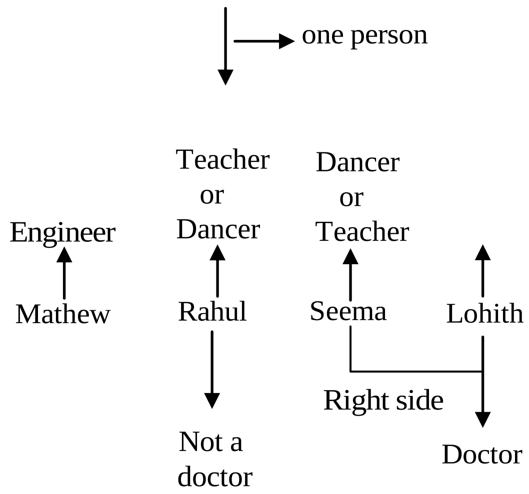
1. Mathew is not standing next to Seema
2. There are two people standing beween Lohit and the engineer
3. Rahul is not a doctor
4. The teacher and the dancer are standing next to each other.
5. Seema is turning to her right to speak to the doctor standing next to her.

Who amongst them is an engineer?

(A) Rahul (B) Mathew (C) Seema (D) Lohit

Key: (B)

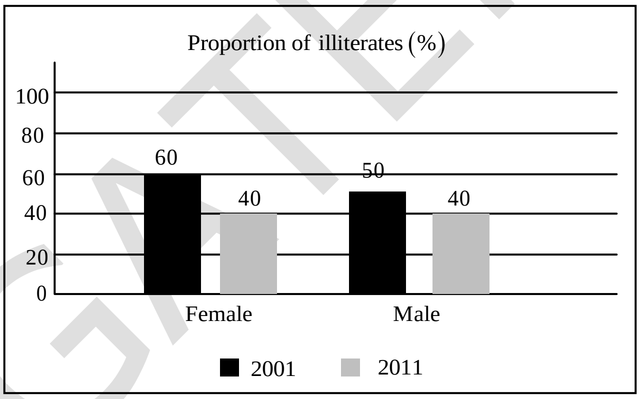
Sol: According to the given data; we have



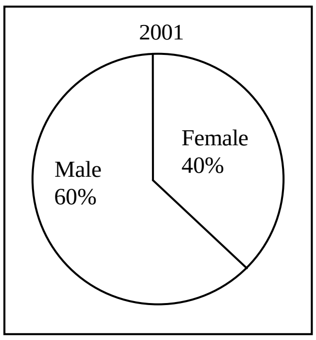
∴ Mathew must be an Engineer.

10. The bar graph in Panel (a) shows the proportion of male and female illiterates in 2001 and 2011. The proportions of males and females in 2001 and 2011 are given in Panel (b) and (c), respectively. The total population did not change during this period.

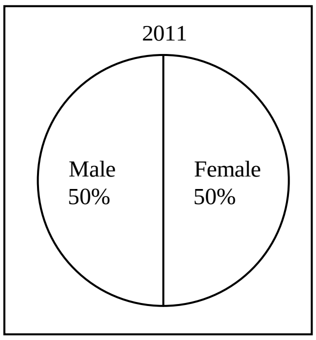
The percentage increase in the total number of literate from 2001 to 2011 is _________.



Panel (a)



Panel (b)



Panel (c)

(A) 33.43 (B) 35.43 (C) 34.43 (D) 30.43

Key: (D)

Sol: Let us assume, that population = 100 [2001–2011]

From panel (a);	From panel (b & c);
Percentage of male literates in 2001 = 50%	Number of males in 2001=60
Percentage of female literates in 2001 = 40%	Number of females in 2001=40
Percentage of male literates in 2011 = 60%	Number of males in 2011=50
Percentage of female literates in 2011 = 60%	Number of females in 2011=50

$\therefore$ Number of male literates in 2001 $= 60 \times \frac{50}{100} = 30$

Number of female literates in 2001 $= 40 \times \frac{40}{100} = 16$

Number of male literates in 2011 $= 50 \times \frac{60}{100} = 30$

Number of female literates in 2011 $= 50 \times \frac{60}{100} = 30$

Total number of literates in 2001 = 30 + 16 = 46

Total number of literates in 2011 = 30 + 30 = 60

$\therefore$ Percentage increase in the total number of literates from 2001–2011

$$= \left[\frac{60-46}{46} \times 100\right]\% = \left[\frac{14}{46} \times 100\right]\% = 30.43\%$$

46 60

ELECTRONICS & COMMUNICATION ENGINEERING

Q. No. 1 to 25 Carry One Mark Each

1. Radiation resistance of a small dipole current element of length l at a frequency of 3 GHz is 3 ohms. If the length is changed by 1%, then the percentage change in the radiation resistance, rounded off two decimal places, is __________%.

Key: (2)

Sol: Given, $R_{rad} = 3\Omega$, and $f = 3\text{GHz}$

% change in length $\frac{d\ell}{\ell} \times 100\% = 1\%$, $\frac{dR_{rad}}{R_{rad}} \times 100\% = ?$

$$R_{rad} = \frac{80\pi^2}{\lambda^2}\ell^2 \qquad \ldots(1)$$

Change in resistance with respect to change in length is $= \frac{dR_{rad}}{d\ell} = 2\ell\left(\frac{80\pi^2}{\lambda^2}\right) \ldots(2)$

$$\frac{\text{Equation}(2)}{\text{Equation}(1)} = \frac{\frac{dR_{rad}}{d\ell}}{R_{rad}} = \frac{2\ell}{\ell^2} = \frac{2}{\ell}$$

$$\Rightarrow \frac{dR_{rad}}{R_{rad}} = 2 \times \frac{d\ell}{\ell}$$

$$\Rightarrow \frac{dR_{rad}}{R_{rad}} \times 100\% = 2 \times \left(\frac{\partial \ell}{\ell} \times 100\%\right) \rightarrow \text{\% change in length} \qquad \left[\because \frac{d\ell}{\ell} \times 100\% = 1\%\right]$$

$\therefore$ % change in radiation resistance = 2%.

2. Which one of the following functions is analytic over the entire complex plane?

(A) $\ell n(z)$ (B) $\cos(z)$ (C) $e^{1/z}$ (D) $\frac{1}{1-z}$

Key: (B)

Sol: $\ell n(z)$ is not analytic at z = 0;

$e^{\frac{1}{z}}$ is not analytic at z = 0 and

$\frac{1}{1-z}$ is not analytic at z = 0.

But cos(z) is analytic over the entire complex plane,

Since $\cos z = \cos(x+iy) = \cos x \cos(iy) - (\sin x \sin(iy))$

$= \cos x \cosh y - i \sin x \sinh y; \rightarrow u + iv \text{ form}$

Where, $u(x,y) = \cos x \cosh y;\ v(x,y) = -\sin x \sinh y$

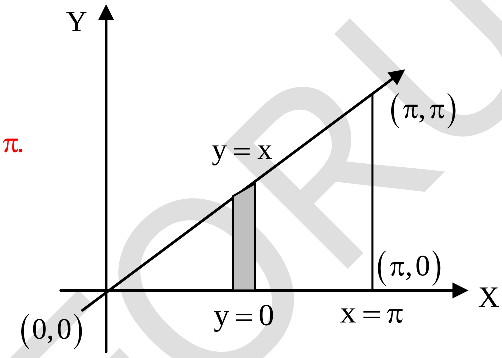
$$\begin{array}{l|l} u_x = -\sin x.\cosh y & v_x = -\cos x.\sinh y \\ u_y = \cos x \sinh y & v_y = -\sin x.\cosh y \end{array}$$

$\therefore\ u_x = v_y \ \&\ u_y = -v_x \ \forall\ z.$

3. The value of the integral $\int_0^{\pi}\int_y^{\pi} \frac{\sin x}{x} dx\, dy,$ is equal to ________.

Key: (2)

Sol: $\int_0^{\pi}\int_0^{\pi} \frac{\sin x}{x} dx\, dy$



Given limits are $x = y \to x = \pi \ \&\ y = 0 \to y = \pi.$

For change of order of integration,

Consider a strip parallel to Y-axis. Then

Limits of y are: $y = 0 \to y = x$ and

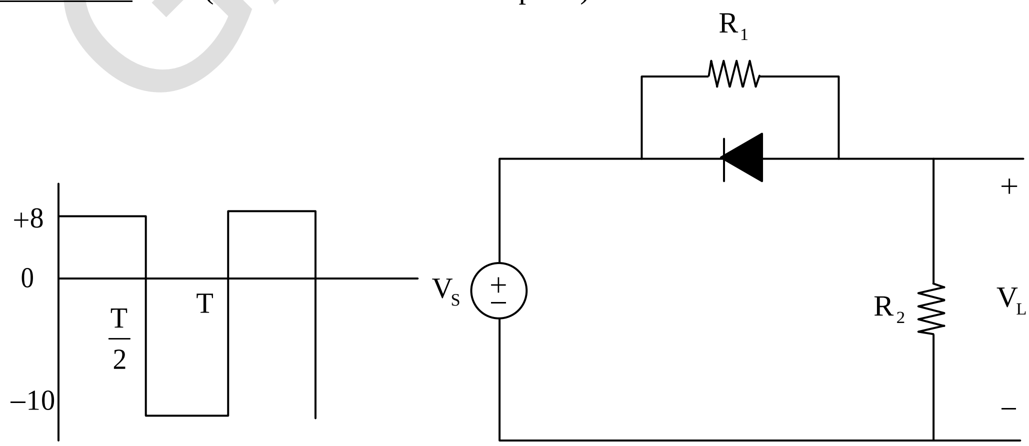
Limits of x: $x = 0 \to x = \pi$

$$\therefore \int_0^{\pi}\int_0^{\pi} \frac{\sin x}{x} dxdy = \int_{x=0}^{\pi}\left[\int_{y=0}^{\pi} \frac{\sin x}{x}.dy\right]dx = \int_{x=0}^{\pi} \frac{\sin x}{x}[y]_0^x.dx = \int_{x=0}^{\pi} \sin x.dx$$

$$= [-\cos x]_0^{\pi} = -[\cos \pi - \cos 0] = -[-1-1] = 2$$

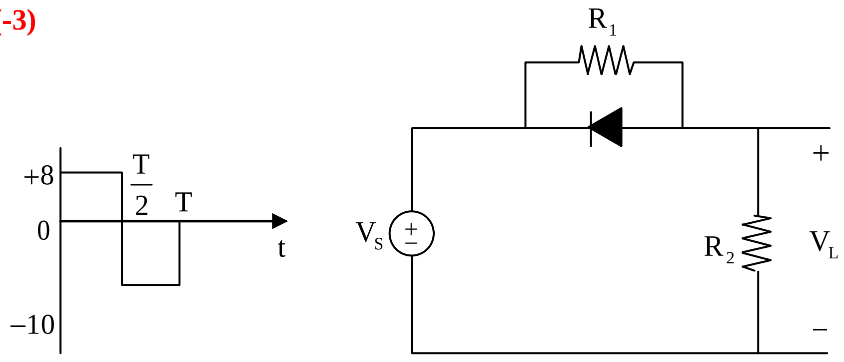
$$\Rightarrow \int_0^{\pi}\int_y^{\pi} \frac{\sin y}{x} dx\, dy = 2$$

4. In the circuit shown, V_S is a square wave of period T with maximum and minimum values of 8V and –10V, respectively, Assume that the diode is ideal and $R_1 = R_2 = 50\Omega$. The average value of V_L is __________ volts (rounded off to 1 decimal place).

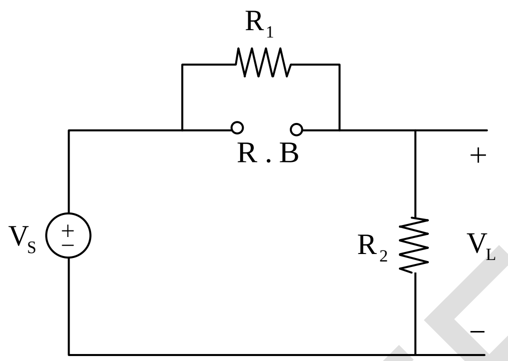


Key: (-3)

Sol:



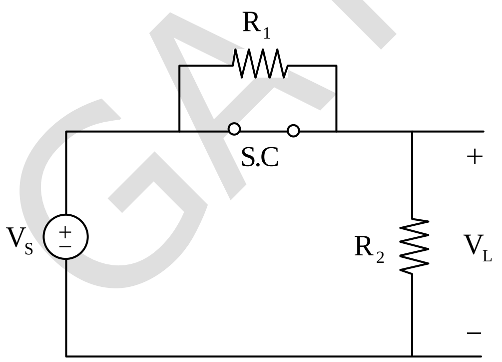
Case (i) For +ve half cycle diode is reverse bias:



$$V_L = \frac{R_2}{R_1 + R_2}.V_S = \frac{50}{100} \times 8V = 4V$$

Case (ii):

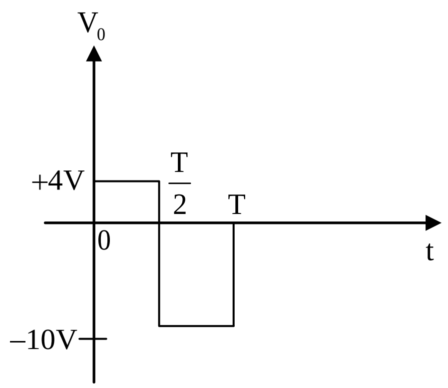
For negative half cycle diode is forward bias and ideally act as short circuit.



$V_L = V_s = -10V.$ So, output waveform is

$$V_{avg} = \frac{4V \times \frac{T}{2} + \left(-10V \times \frac{T}{2}\right)}{T} = 2V - 5V = -3V$$

$\therefore V_{avg} = -3V$



5. The number of distinct eigen values of the matrix $A=\begin{bmatrix}2&2&3&3\\0&1&1&1\\0&0&3&3\\0&0&0&2\end{bmatrix}$ is equal to ________.

Key: (3)

Sol: Given,

$$\text{Matrix } A=\begin{bmatrix}2&2&3&3\\0&1&1&1\\0&0&3&3\\0&0&0&2\end{bmatrix}\to \text{Upper triangular matrix}$$

$\therefore$ Eigen values of A are $\lambda=2,1,3,2\left[\text{diagonal elements}\right]$

$\therefore$ The number of distinct eigen values of the matrix = 3.

6. The families of curves represented by the solution of the equation $\frac{dy}{dx}=-\left(\frac{x}{y}\right)^n$

For n = –1 and n = +1, respectively, are

(A) Hyperbolas and Circles
(B) Circles and Hyperbolas
(C) Hyperbolas and Parabolas
(D) Parabolas and Circles

Key: (A)

Sol: Given D.E $\frac{dy}{dx}=-\left(\frac{x}{y}\right)^n$

$$\Rightarrow \frac{dy}{dx}=-\frac{x^n}{y^n} \qquad \ldots(1)$$

For n = –1; we have

$$\frac{dy}{dx}=-\frac{x^{-1}}{y^{-1}}\left[\because(1)\right]$$

$$\Rightarrow\frac{dy}{dx}=-\frac{y}{x}\Rightarrow\frac{1}{y}dy=\frac{-1}{x}dx\left[\text{variable separable D.E}\right]$$

Integrating on both sides; we get

$$\Rightarrow \ell ny=-\ell nx+\ell nc\Rightarrow \ell n(xy)=\ell nc$$

$$\Rightarrow xy=c^2$$

$\therefore\ xy=c^2\to$ represents family of hyperbolas.

For n =1; we have

$$\frac{dy}{dx}=-\frac{x}{y} \qquad \left[\because(1)\right]$$

$\Rightarrow ydy = -xdx \to$ variable$-$separable D.E

Integrating on bothsides; we get

$$\frac{y^2}{2} = -\frac{x^2}{2} + c \Rightarrow \frac{x^2}{2} + \frac{y^2}{2} = c$$
$$\Rightarrow x^2 + y^2 = c$$
$$\Rightarrow x^2 + y^2 = \left(\sqrt{c}\right)^2 \text{; which represents family of circles}$$

7. In the table shown, List-I and List-II, respectively, contain terms appearing on the left-hand side and the right-hand side of Maxwell's equations (in their standard form). Match the left-hand side with the corresponding right-hand side.

List-I	List-II
1. $\nabla.D$	(P) 0
2. $\nabla \times E$	(Q) ρ
3. $\nabla.B$	(R) $-\frac{\partial B}{\partial t}$
4. $\nabla \times H$	(S) $J + \frac{\partial D}{\partial t}$

(A) 1-Q, 2-R, 3-P, 4-S (B) 1-Q, 2-S, 3-P, 4-R

(C) 1-P, 2-R, 3-Q, 4-S (D) 1-R, 2-Q, 3-S, 4-P

Key: (A)

Sol: $\nabla.D = \rho$

$$\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t}$$

$$\nabla.\bar{B} = 0$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$\therefore 1-Q, 2-R, 3-P, 4-S \to$ Option (A) is correct

8. Consider the signal $f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$, where t is in seconds. Its fundamental time period, in seconds, is ___________.

Key: (12)

Sol: It is given that,

$$f(t) = 1 + 2\cos\pi t + 3\sin\frac{2\pi}{3}t + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$$

Its fundamental frequency ω_0

$$\omega_0 = \frac{HCF(\pi, 2\pi, \pi)}{LCM(1,3,2)} = \frac{\pi}{6}$$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi \times 6}{\pi} = 12\text{ sec}$$

9. If X and Y are random variables such that $E[2X+Y]=0$ and $E[X+2Y]=33$, the $E[X]+E[Y] =$ __________.

Key: (11)

Sol: Given, $E[2x+y]=0$ & $E[x+2y]=33$

$\Rightarrow 2E(x)+E(y)=0 \ldots(1)$ $\Rightarrow E(x)+2E(y)=33 \ldots(2)$

Solving (1) and (2); we have

$$2E(x)+E(y)=0$$
$$\underline{\overset{-}{2E(x)}+\overset{-}{4E(y)}=\overset{-}{66}} \quad [\because \text{equation }(2)\text{ multiplied by }2]$$

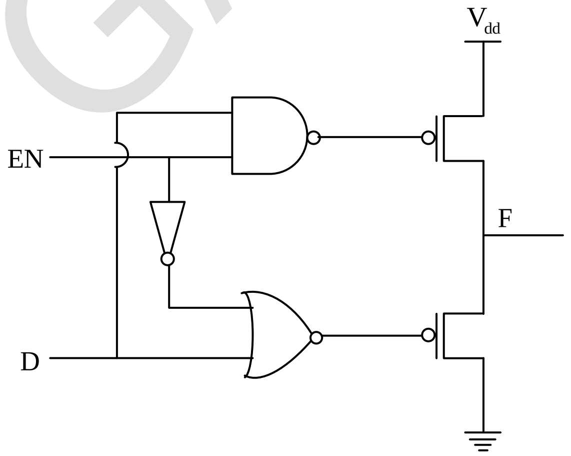
$$-3E(y)=-66 \Rightarrow E(y)=22$$

From (1);

$$2E(x)+E(y)=0 \Rightarrow 2E(x)=-E(y)$$
$$\Rightarrow 2E(x)=-22 \Rightarrow E(x)=-11$$

$$\therefore E[x]+E[y]=-11+22=11 \Rightarrow E(x)+E(y)=11$$

10. In the circuit shown, what are the values of F for EN = 0 and EN = 1, respectively?

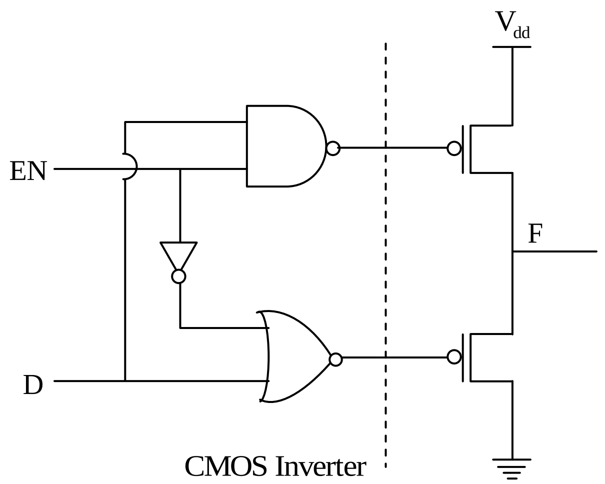


(A) 0 and 1 (B) Hi – Z and $\bar{D}$

(C) 0 and D (D) Hi – Z and D

Key: (D)

Sol:



→ NAND gate enabled, when their enable input is "1" and NOR gate enabled, when their enable input is "0".

Case (i):

When EN = 0, the both the logic gates NAND and NOR disabled, so CMOS inverter input is floating. So, output is also high impedance state.

$F = Hi - z$

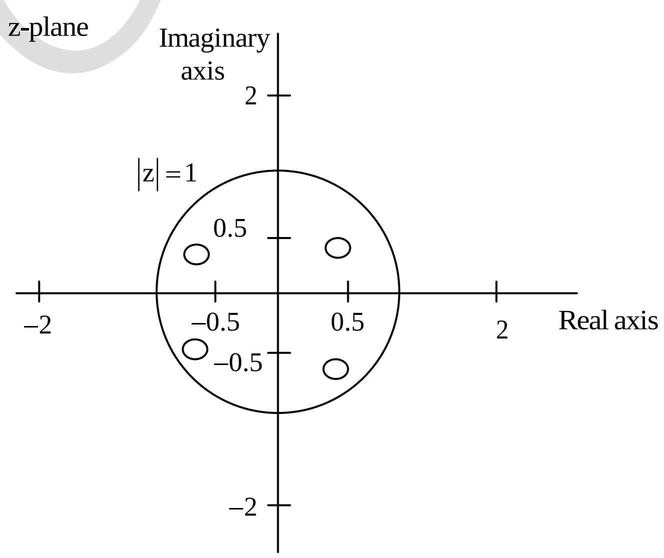
Case (ii):

When EN = 1, then both the logic gates NAND and NOR are enabled with output $\bar{D}$ that is input of CMOS inverter.

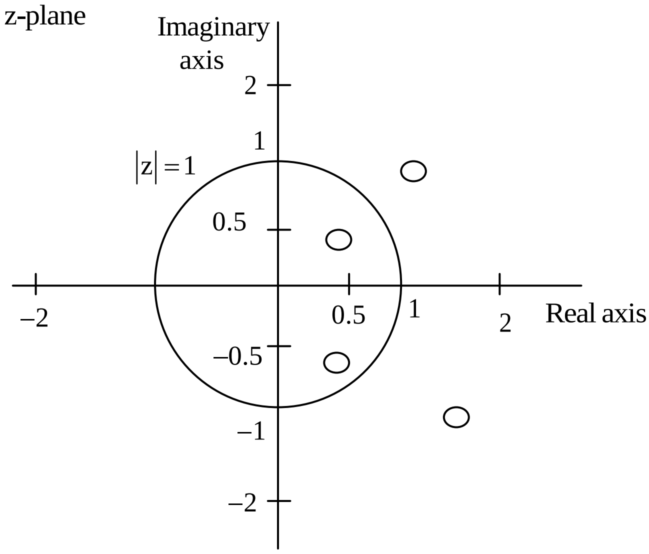
So, $F = \bar{\bar{D}} = D$

11. Let H(z) be the z-transform of a real-valued discrete time signal h[n]. If $P(z) = H(z)H\left(\frac{1}{z}\right)$ has a zero $z = \frac{1}{2} + \frac{1}{2}j$, and P(z) has a total of four zeros, which one of the following plots represents all the zeros correctly?

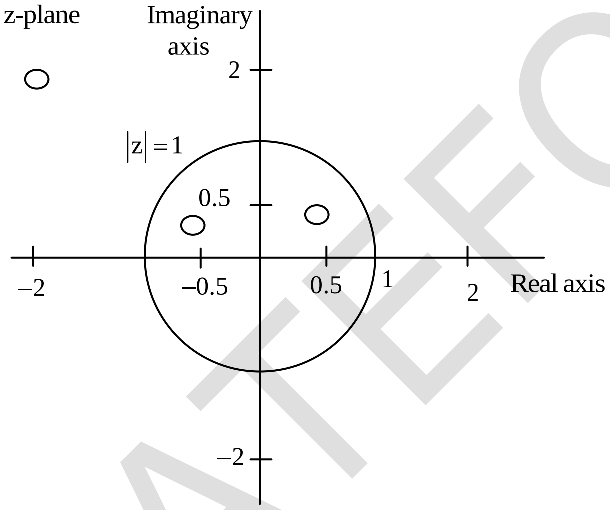
(A)



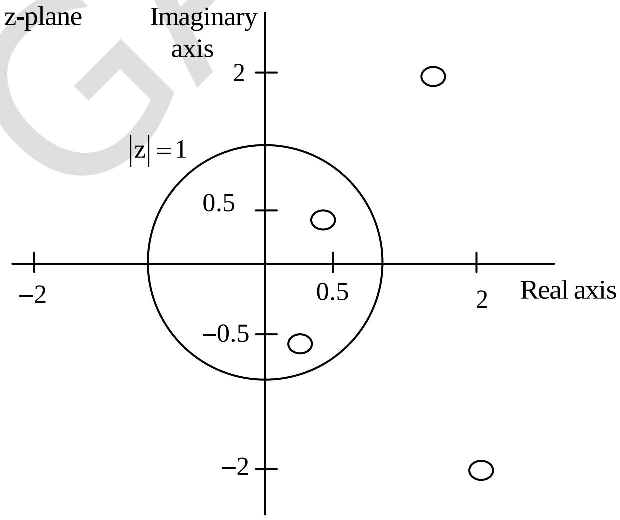
(B)


z-plane
Imaginary axis
2
1
|z| = 1
0.5
−2
0.5
1
2
Real axis
−0.5
−1
−2

(C)


z-plane
Imaginary axis
2
|z| = 1
0.5
−2
−0.5
0.5
1
2
Real axis
−2

(D)


z-plane
Imaginary axis
2
|z| = 1
0.5
−2
0.5
2
Real axis
−0.5
−2

Key: (B)

Sol: It is given that H(z) is z-transform of a real-valued signal h(n).

$P(z)=H(z)H\left(\frac{1}{z}\right)$ and $P(z)$ has 4 zeros we can say zeros of $P(z)$ are, sum of zeros of H(z) and zeros of $H\left(\frac{1}{z}\right)$

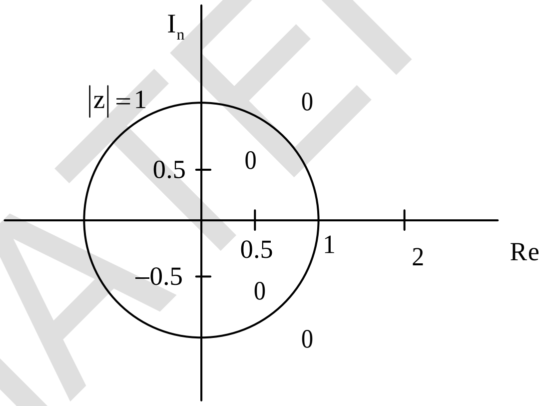
If $z_1=\frac{1}{2}+j\frac{1}{2}$ is one zero then there must be a zero at $z_1^*=\frac{1}{2}-j\frac{1}{2}$

Let z_1, z_1^* represent zeros of H(z) then the zeros of $H\left(\frac{1}{z}\right)$ will be $\frac{1}{z_1}$ and $\left(\frac{1}{z_1}\right)^*$

$$\frac{1}{z_1}=\frac{1}{\frac{1}{2}+j\frac{1}{2}}=1-i$$

$$\left(\frac{1}{z_1}\right)^*=1+j$$

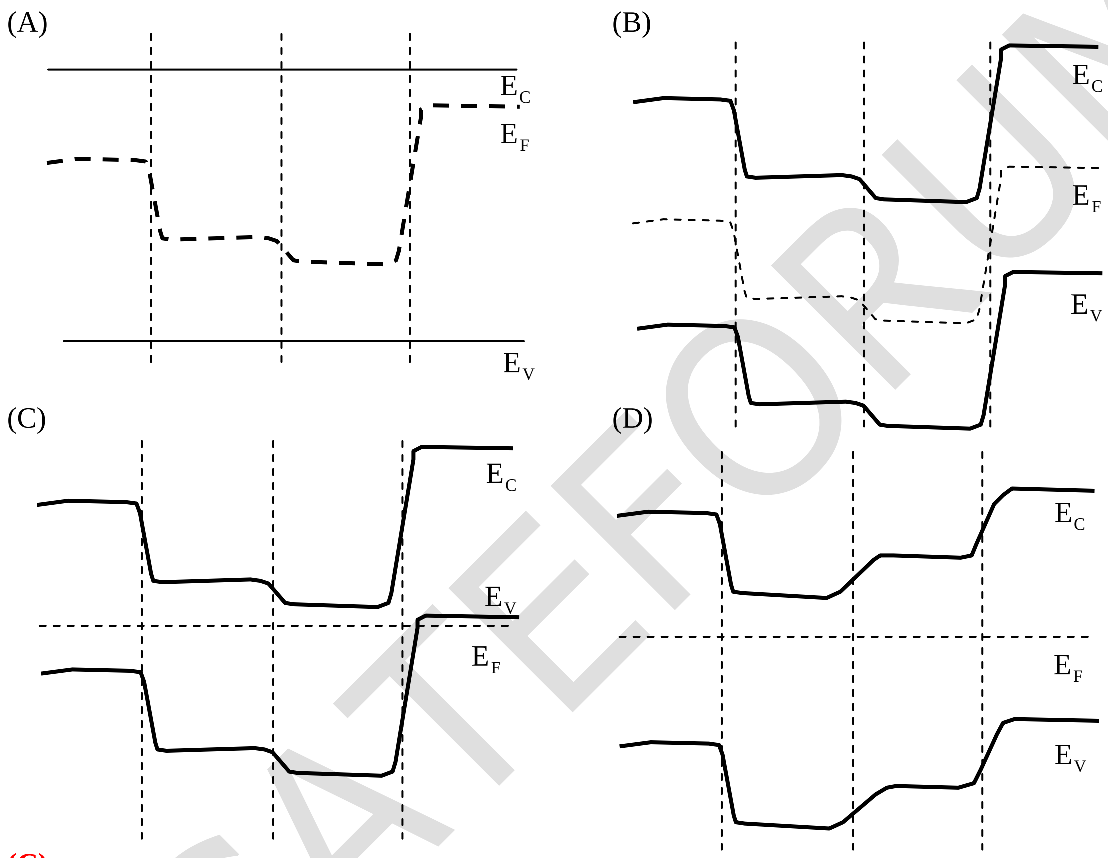
So the 4 zeros of P(z) are $\left(\frac{1}{2}\pm j\frac{1}{2}\right)$ and $(1\pm j)$ or $\left[0.707\underline{|\pm45^\circ}\right]$ and $\left[\sqrt{2}\underline{|\pm45^\circ}\right]$



Even option D looks like similar but in option B, the zeros that are outside the unit circle have real part 2, but we need 1.

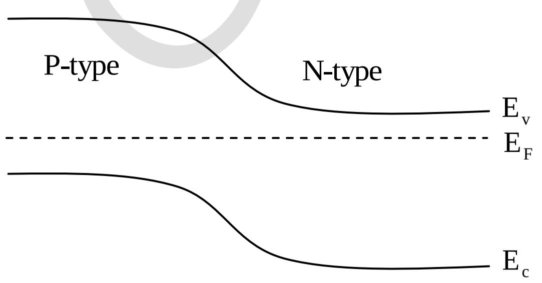
12. Which one of the following options describes correctly the equilibrium band diagram at T= 300 K of a Silicon $pnn^{+}p^{++}$ configuration shown in the figure?

p	n	n^{+}	p^{++}



Key: (C)

Sol: Typical energy band diagram of P-N junction diode:

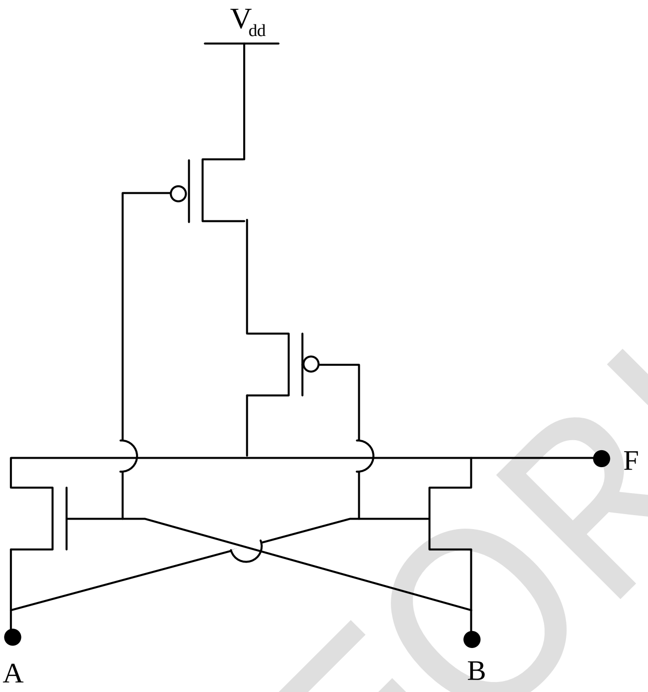


- First of all of equilibrium, Fermi energy level (E_F) is constant
- For P-type of semiconductor

$$E_F = E_v - kT\ell n\left(\frac{N_A}{N_V}\right) \quad (\text{For } N_A < N_V)$$

- For $pnn^{+}p^{++}$ configuration, Fermi-energy level (E_F) is more closer to E_V compare to E_c.

13. In the circuit shown, A and B are the inputs and F is the output. What is the functionality of the circuit?

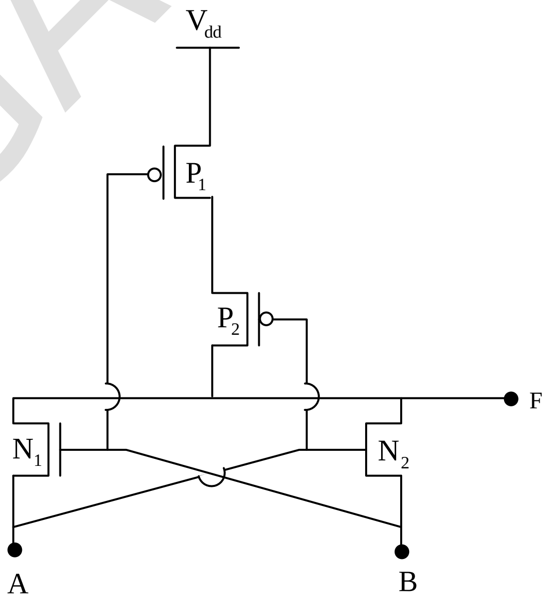


(A) XOR (B) XNOR (C) Latch (D) SRAM cell

Key: (B)

Sol: NMOS behaves as ON switch for logic 1.

PMOS behaves as ON switch for logic 0.

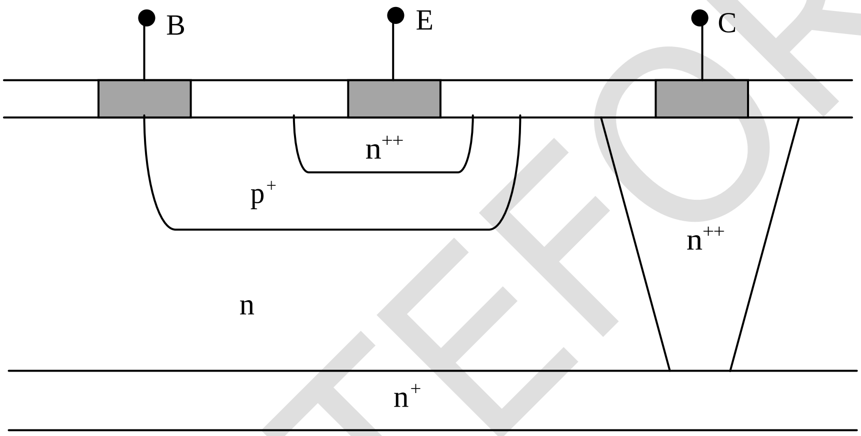


Let make the truth table

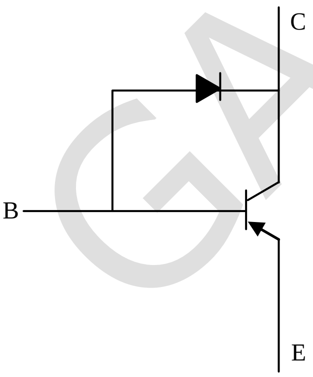
A	B	P_1	N_1	P_2	N_2	F	Reasons
0	0	0	0	0	0	1	[$\because$ P_1, P_2 on V_{dd} is will be as F
0	1	1	1	0	0	0	[$\because$ N_1 is ON, So F = A=0]
1	0	0	0	1	1	0	[$\because$ N_2 IS ON, SO F = B= 0]
1	1	1	1	1	1	1	[$\because$ N_1, N_2 ON, So, F = A= B = 1]

So from the above truth table we can say $F = A \odot B$

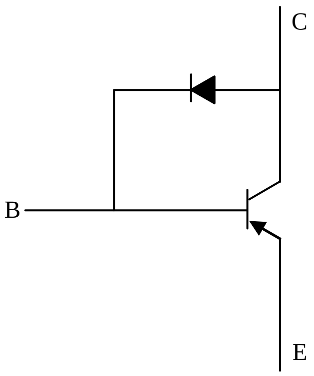
14. The correct circuit representation of the structure shown in the figure is



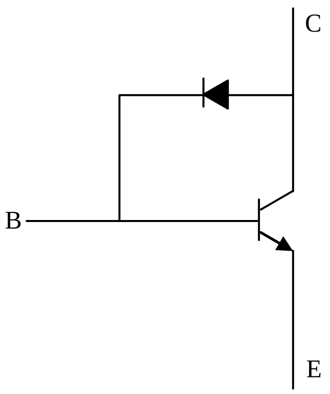
(A)



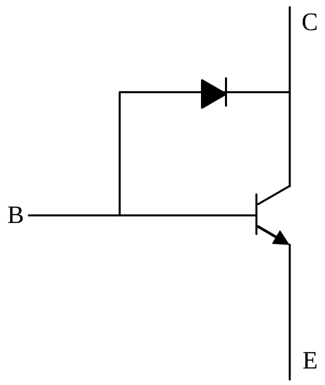
(B)



(C)

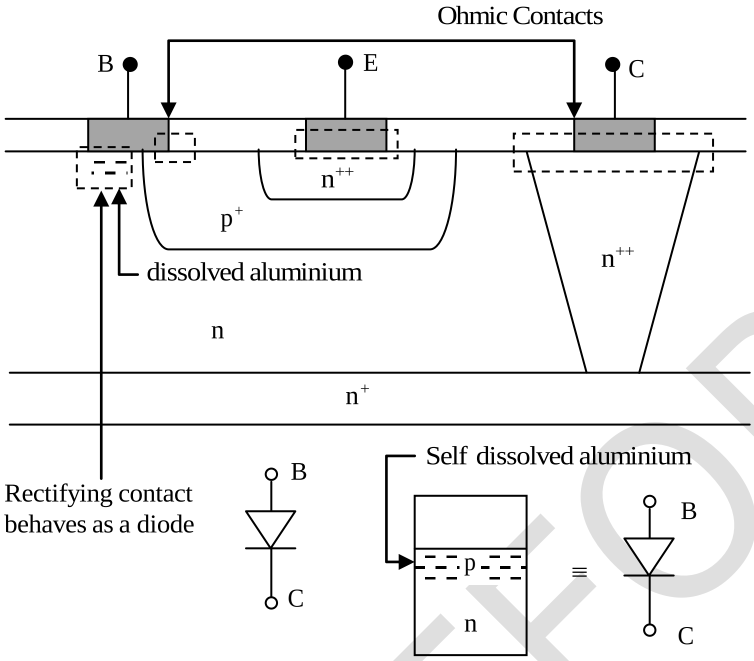


(D)

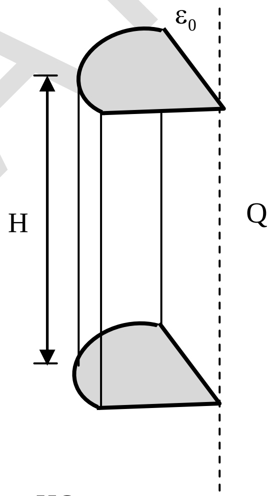


Key: (D)

Sol: Aluminum metal in contact with a lightly doped n type silicon forms a non ohmic rectifying contact because the trivalent aluminium easily dissolves into silicon and convert the part of n type semiconductor into p type.



15. What is the electric flux $\left(\int \vec{E}.d\hat{a}\right)$ through a quarter-cylinder of height H (as shown in the figure) due to an infinitely long the line charge along the axis of the cylinder with a charge density of Q?



(A) $\dfrac{4H}{Q\varepsilon_0}$ (B) $\dfrac{HQ}{4\varepsilon_0}$ (C) $\dfrac{HQ}{\varepsilon_0}$ (D) $\dfrac{H\varepsilon_0}{4Q}$

Key: (B)

Sol: The total electric flux leaving the total cylinder of height "H" is

$$=\oiint \bar{E}.d\bar{A}=\frac{QH}{\epsilon_0}$$

$\therefore$ The electric through a quarter cylinder of height "H" is

$$=\frac{\oiint \bar{E}.d\bar{A}}{4}=\frac{QH}{4\epsilon_0}$$

$\therefore$ Option (B) is correct.

16. Let Z be an exponential random variable with mean 1. That is, the cumulative distribution function of Z is given by

$$F_z(x)=\begin{cases}1-e^{-x} & \text{if } x\ge 0\\ 0 & \text{if } x<0\end{cases}$$

The Pr(Z > 2| Z > 1), rounded off to two decimal places, is equal to _________.

Key: (0.37)

Sol: Given,

Cumulative distribution function, $F_z(x)=\begin{cases}1-e^{-x} & \text{if } x\ge 0\\ 0 & \text{if } x<0\end{cases}$

$$\Rightarrow F'_z(x)=f_z(x)=\begin{cases}e^{-x}; & x\ge 0\\ 0; & x<0\end{cases}\rightarrow \text{probability density function}$$

Where Z be an exponential R.V with mean '1'

$$\therefore \Pr\left[\frac{z>2}{z>1}\right]=\frac{\Pr[z>2\cap z>1]}{\Pr[z>1]}\quad [\text{Using conditional probability}]$$

$$=\frac{\Pr[z>2]}{\Pr[z>1]}=\frac{\int_2^\infty f(x)dx}{\int_1^\infty f(x)dx}=\frac{\int_0^\infty e^{-x}dx}{\int_1^\infty e^{-x}.dx}$$

$$\Rightarrow \Pr\left[\frac{z>2}{z>1}\right]=\frac{\left[-e^{-x}\right]_2^\infty}{\left[-e^{-x}\right]_1^\infty}=\frac{\left[0+e^{-2}\right]}{\left[0+e^{-1}\right]}=\frac{e^{-2}}{e^{-1}}=\frac{1}{e}\approx 0.37$$

17. A linear Hamming code is used to map 4-bit messages to 7-bit codewords. The encoder mapping is linear. If the message 0001 is mapped to the codeword 0000111, and the message 0011 is mapped to the codeword 1100110, then the message 0010 is mapped to

(A) 0010011 (B) 1111111 (C) 1111000 (D) 1100001

Key: (D)

Sol: Message (1) $\Rightarrow$ 0001

Message (2) $\Rightarrow$ 0011

0000111 $\Rightarrow$ Codeword (1)

1100110 $\Rightarrow$ Codeword (2)

Since it is a linear hamming code,

Message (1) + Message (2) results in codeword (1) + codeword (2)

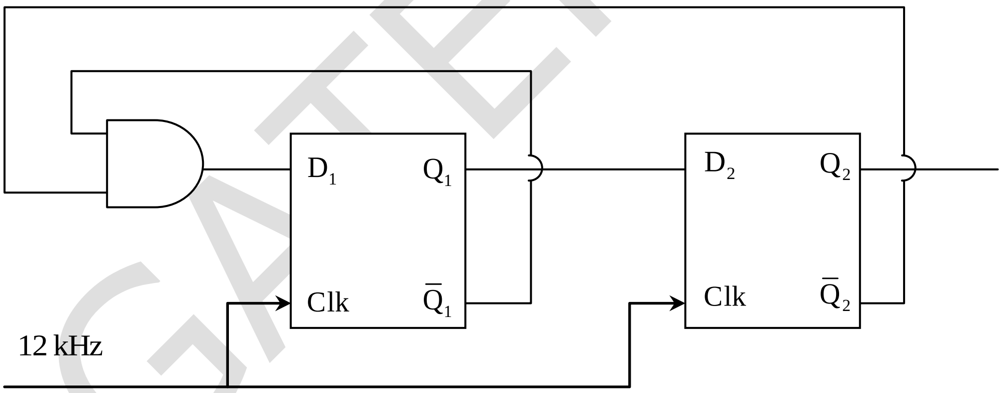
$\Rightarrow$ Addition of binary is logical XOR

$\therefore$ Message | Codeword

$$\begin{array}{r} 0001 \\ \oplus\, 0011 \\ \hline 0010 \end{array} \qquad \begin{array}{r} 0000111 \\ \oplus\, 1100110 \\ \hline 1100001 \end{array}$$

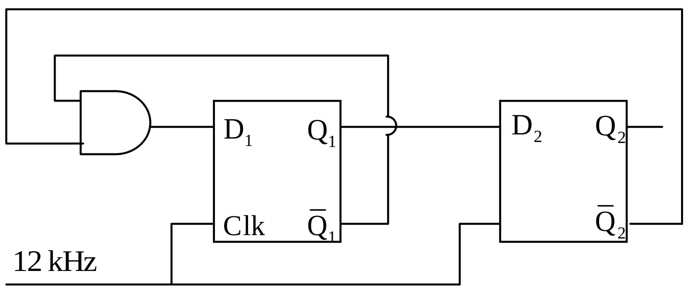
Message (3) | Codeword (3)

18. In the circuit shown, the clock frequency, i.e., the frequency of the Clk signal, is 12 kHz. The frequency of the signal at Q_2 is __________ kHz.



Key: (4)

Sol:



$D_2(Q_1)$	$D_1(\bar{Q}_1 . \bar{Q}_2)$	Q_2	Q_1	
–	–	0	0	← Initially
0	1	0	1	1^{st} clock
1	0	1	0	2^{nd} clock
0	0	0	0	3^{rd} clock

$Q_2Q_1 \rightarrow 00 \rightarrow 01 \rightarrow 10 \rightarrow 00$

It is a MOD-3 synchronous counter

So, $f_{Q_2} = \frac{f_i}{3} = \frac{12\text{kHz}}{3} = 4\text{ kHz}$

19. A standard CMOS inverter is designed with equal rise and fall times $(\beta_n = \beta_p)$. If the width of the pMOS transistor in the inverter is increased, what would be the effect on the LOW noise margin (NM_L) and the HIGH noise margin NM_H?

(A) NM_L increases and NM_H decrease

(B) Both NM_L and NM_H increase

(C) No change in the noise margins

(D) NM_L decreases and NM_H increases

Key: (A)

Sol: Given, $\beta_n = \beta_p$ for standard CMOS inverter

We know that, standard definition of noise margin

$NM_L = V_{IL} - V_{OLU}$

$MH_H = V_{OHU} - V_{IH}$

Case(i):

$\beta_n = \beta_p$, let take $V_{DD} = 5,\ V_{TN} = -V_{TP} = 0.8,\ V_I = 2.5$

$$V_{IL} = V_{TN} + \frac{3}{8}[V_{DD} + V_{TP} - V_{TN}] = 2.075$$

$$V_{OLU} = \frac{1}{2}[2V_{IH} - V_{DD} - V_{TN} - V_{TP}] = 0.45$$

$$(N_M)_L = V_{IL} - V_{OLU} = 1.625$$

$$V_{OUH} = \frac{1}{2}[2V_{IL} + V_{DD} - V_{TN} - V_{TP}] = 4.575$$

$$V_{IH} = V_{TN} + \frac{5}{8}[V_{DD} + V_{TP} - V_{TN}] = 2.95$$

$$(N_M)_H = 1.625$$

Case (ii): $(\beta_n \neq \beta_p)$ Because width of PMOS increased

$$\left(\frac{W}{L}\right)_P > \left(\frac{W}{L}\right)_n$$

$\frac{\beta_P}{P_n} > 1,\ \frac{\beta_n}{\beta_p} < 1$, Let assume 0.8

$$V_{IL} = V_{TN} + \frac{V_{DD} + V_{TP} - V_{TN}}{\left(\frac{\beta_n}{\beta_p} - 1\right)}\left[2\sqrt{\frac{\frac{\beta_n}{\beta_p}}{\frac{\beta_n}{\beta_p} - 3}} - 1\right] = 2.216$$

$$V_{OLU} = \frac{V_{IH}\left(1 + \frac{\beta_n}{\beta_p}\right) - V_{DD} - \left(\frac{\beta_n}{\beta_p}\right)(V_{TN} - V_{TP})}{2\left(\frac{\beta_n}{\beta_p}\right)}$$

$$= \frac{3.048 \times 1.8 - 5 - 0.64 + 0.8}{2 \times 0.8}$$

$$= 0.404$$

$$(NM)_L = 2.216 - 0.404 = 1.812(\text{Increased})$$

$$V_{0UH} = \frac{1}{2}\left[1 + \left(\frac{\beta_n}{\beta_p}\right)V_{IL} + V_{DD} - \left(\frac{\beta_n}{\beta_p}\right)V_{TN} - V_{TP}\right]$$

$$= \frac{1}{2}\left[(2.216)1.8 + 5 - 0.64 + 0.8\right] = 4.574$$

$$V_{IH} = V_{TN} + \frac{(V_{DD} + V_{TP} - V_{TN})}{\frac{\beta_n}{\beta_p} - 1}\left[\frac{2\frac{\beta_n}{\beta_p}}{\sqrt{3\frac{\beta_n}{\beta_p} + 1}} - 1\right]$$

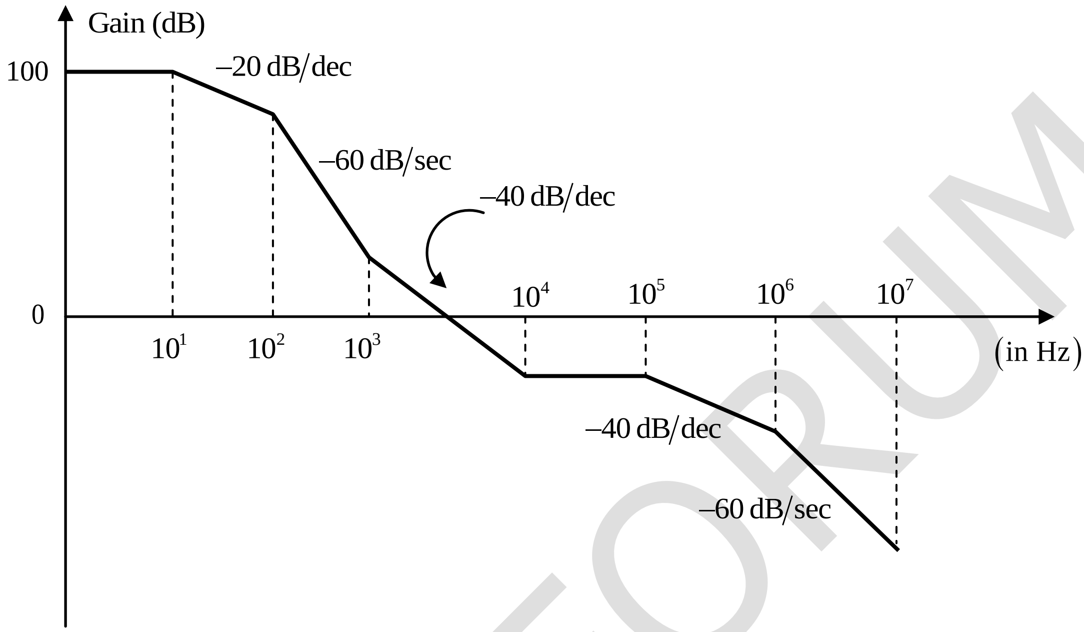
$$= 0.8 + \frac{[5 - 1.6]}{-0.2} \times -0.132$$

$$= 2.248 + 0.8 = 3.048$$

$$(NM)_H = 4.574 - 3.048$$

$$= 1.526(\text{decreased})$$

20. For an LTI system, the Bode plot for its gain is as illustrated in the figure shown. The number of system poles N_p and the number of system zeros N_Z in the frequency range $1\text{Hz} \le f \le 10^7\text{Hz}$ is



(A) $N_p = 4, N_z = 2$ (B) $N_p = 7, N_z = 4$

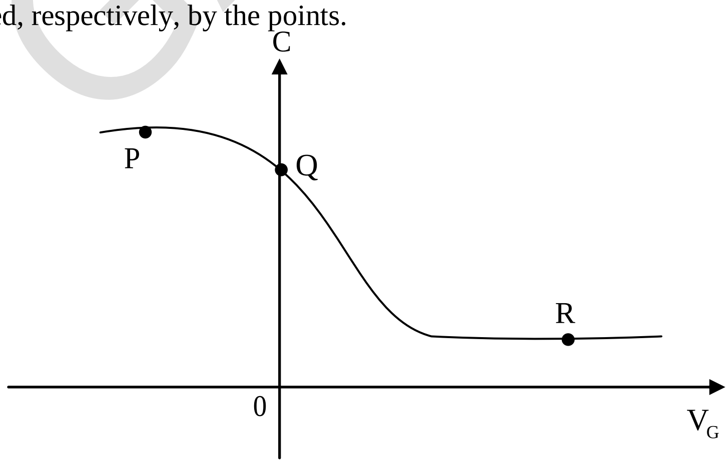
(C) $N_p = 6, N_z = 3$ (D) $N_p = 5, N_z = 2$

Key: (C)

Sol: From the given plot

$N_p = 6, N_z = 3$

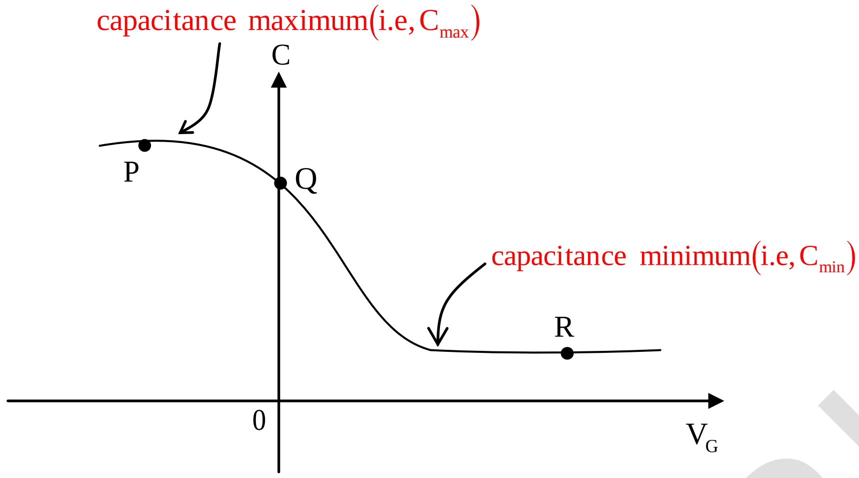
21. The figure shows the high- frequency C-V curve of a MOS capacitor (at T=300 K) with $\Phi_{ms} = 0$ V and no oxide charges. The flat-band, inversion, and accumulation conditions are represented, respectively, by the points.



(A) R,P,Q (B) Q,P,R (C) P,Q,R (D) Q,R,P

Key: (D)

Sol:



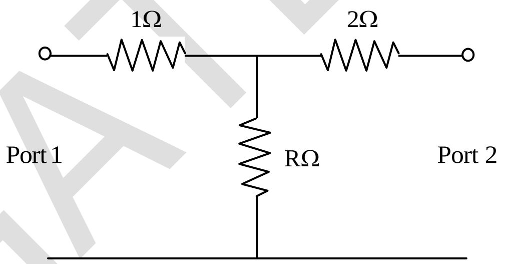
At higher frequency C_{min} minimum capacitance obtained in inversion-regime so point R-belongs to inversion regime.

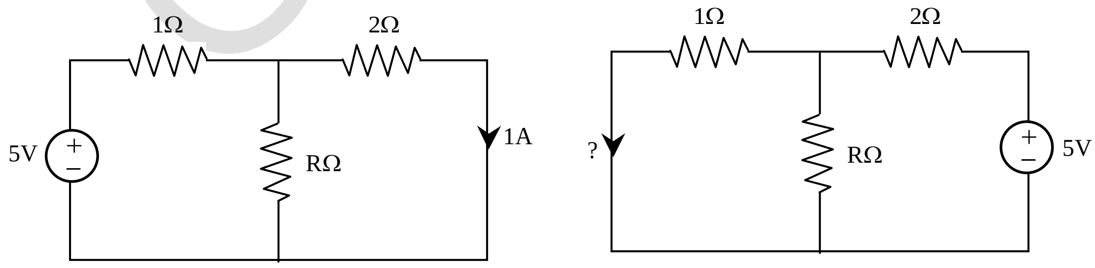
Maximum capacitance obtained in accumulation regime so point p-line is accumulation regime.

For $\phi_m = 0V$, flat band occurs at $V_G = 0V$, so point ϕ lies at flat-band regime.

22. Consider the two-port resistive network shown in the figure. When an excitation of 5V is applied across Port 1, and Port 2 is shorted, the current through the short circuit at Port 2 is measured to be 1 A (see (a) in the figure).

Now, if an excitation of 5 V is applied across port 2, and port 1 is shorted (see (b) in the figure), what is the current through the short circuit at port 1?



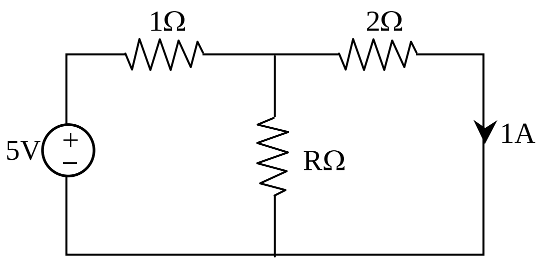


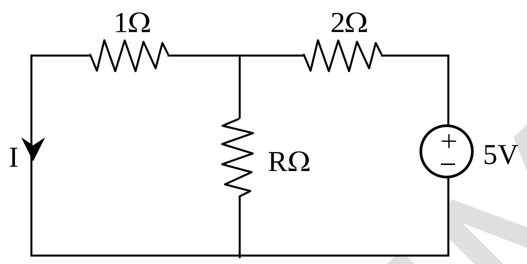
(a) (b)

(A) 1A (B) 2A (C) 2.5A (D) 0.5A

Key: (A)

Sol:





By reciprocity theorem, by referring (figure a),

We can say current I in (figure b) will be 1A. $\left[\frac{V}{I} = k, \frac{V_1}{V_2} = \frac{I_1}{I_2}\right]$

23. Let Y (s) be the unit-step response of a causal system having a transfer function

$$G(s) = \frac{3-s}{(s+1)(s+3)}$$

That is, $Y(s) = \frac{G(s)}{s}$. The forced response of the system is

(A) $u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$ (B) $2u(t)$

(C) $u(t)$ (D) $2u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$

Key: (A)

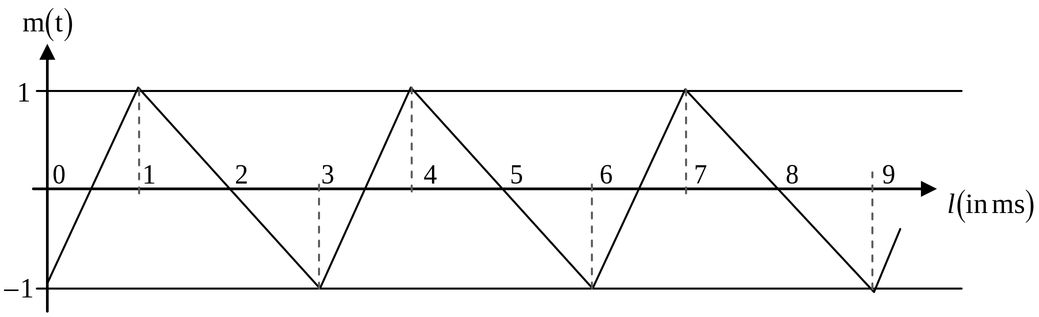
Sol: Forced response is the response, due to external input signal

$$Y(s) = \frac{G(s)}{s} = \frac{3-s}{s(s+1)(s+3)}$$

$$= \frac{1}{s} + \frac{-2}{s+1} + \frac{1}{s+3}$$

$$\Rightarrow y(t) = u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$$

24. The baseband signal m(t) shown in the figure is phase-modulated to generate the PM signal $\varphi(t) = \cos(2\pi f_c t + km(t))$.



The time t on the x-axis in the figure is in milliseconds. If the carrier frequency is $f_c = 50$ kHz and $k = 10\pi$, the ratio of the minimum instantaneous frequency (in kHz) to the maximum instantaneous frequency (in kHz) is_____ (rounded off to 2 decimal places).

Key: (0.75)

Sol: Instantaneous frequency $(f_i) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$

$$\theta(t) = 2\pi f_c t + km(t)$$

$$\therefore f_i = \frac{1}{2\pi}\frac{d}{dt}\left[2\pi f_c t + km(t)\right] = f_c + \frac{K}{2\pi}\frac{dm}{dt}$$

$$f_{i_{max}} = f_c + \frac{K}{2\pi}\frac{dm}{dt}\bigg|_{max} \; ; \quad f_{i_{min}} = f_c + \frac{K}{2\pi}\frac{dm}{dt}\bigg|_{min}$$

$$\frac{dm}{dt}\bigg|_{max} = 2 \quad \frac{dm}{dt}\bigg|_{min} = -1$$

$$f_{i_{max}} = 50K + (5)(2)K = 60K$$

$$f_{i_{min}} = (50K) + (5)(-1)K = 45K$$

$$\frac{f_{i_{min}}}{f_{i_{max}}} = \frac{45K}{60K} = \frac{3}{4} = 0.75$$

25. The value of the contour integral

$$\frac{1}{2\pi j}\oint\left(z + \frac{1}{z}\right)^2 dz$$

Evaluated over the unit circle $|z| = 1$ is__________.

Key: (0)

Sol: $$\frac{1}{2\pi j}\oint\left(z + \frac{1}{z}\right)^2 dz = \frac{1}{2\pi j}\oint\frac{(z^2+1)^2}{z^2}dz \qquad ...(1)$$

$\therefore$ Singular point z=0; which lies inside unit circle $|z| = 1$

$\therefore$ Using Cauchy's generalization integration formula, we have

$$\oint_c \frac{f(z)}{(z - z_0)^n}dz = 2\pi j\frac{\left[f^{n-1}(z_0)\right]}{(n-1)!}$$

$$\Rightarrow \therefore \oint_C \frac{(z^2+1)^2}{(z-0)^2}dz = 2\pi j\frac{\left[(z^2+1)^2\right]'}{1!}\Bigg/_{z=0}; \quad ' \to \text{denotes first derivative}$$

$$= 2\pi j\left[2(z^2+1)(2z)\right]\Big/_{z=0} = 2\pi j[0] = 0.$$

From (1),

$$\frac{1}{2\pi j}\oint\left(z + \frac{1}{z}\right)^2 dz = \frac{1}{2\pi j}\oint\frac{(z^2+1)^2}{(z-0)^2} = \frac{1}{2\pi j}(0) = 0$$

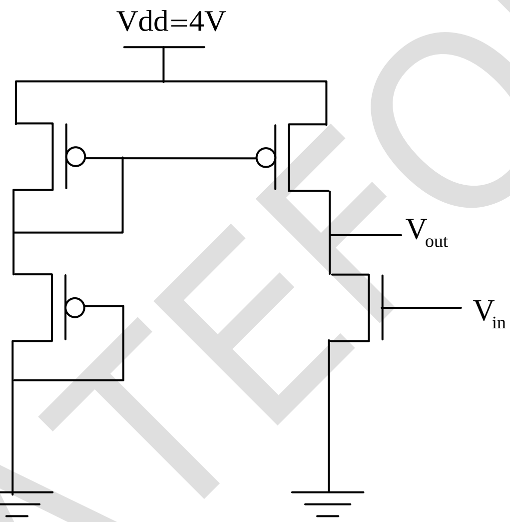
Q. No. 26 - 55 Carry Two Marks Each

26. In the circuit shown, the threshold voltages of the pMOS $\left(\left|V_{tp}\right|\right)$ and nMOS $\left(V_{tn}\right)$ transistors are both equal to 1 V. All the transistors have the same output resistance r_{ds} of $6\,M\Omega$. The other parameters are listed below:

$$\mu_n C_{ox} = 60\,\mu A/V^2;\left(\frac{W}{L}\right)_{nMOS} = 5$$

$$\mu_p C_{ox} = 30\,\mu A/V^2;\left(\frac{W}{L}\right)_{pMOS} = 10$$

μ_n and μ_p are the carrier mobilities, and C_{ox} is the oxide capacitance per unit area. Ignoring the effect of channel length modulation and body bias, the gain of the circuit is______ (rounded off to 1 decimal place).



Key: (-900)

Sol: M 1 and M2 will have equal current flowing also since they are identical $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$

$\therefore V_{SG_1} = V_{SG_2}$ also, By KVL in loop

$4 = V_{SG_1} + V_{SG_2}$

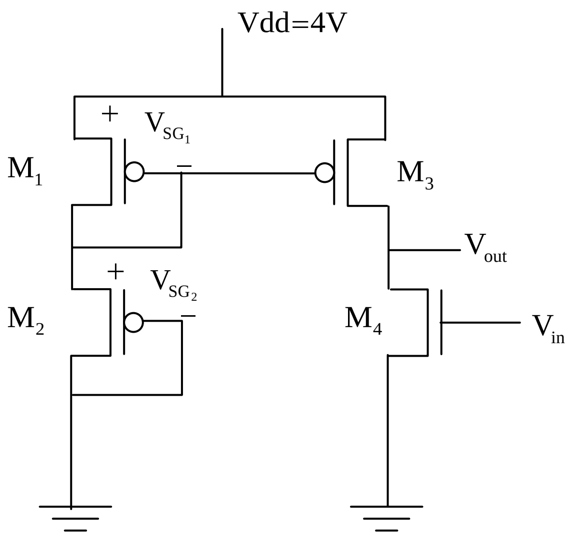
$\therefore V_{SG_1} = V_{SG_2} = 2V$

$\therefore$ current through M_1 and M_2

$$I = \left(\frac{1}{2}\right)\left(\mu_p C_{ox}\right)\left(\frac{W}{L}\right)\left[V_{SG} - \left|V_T\right|\right]^2$$

$$= \left(\frac{1}{2}\right)(30)(10)(2-1)^2 = 150\,\mu A$$

M_1 and M_3 are matched with same



$\left(\frac{W}{L}\right)$ and same V_{SG} hence,

$I_{M_3} = I_{M_4} = 150\mu A$

For MOSFET M_4,

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$$

$$= \sqrt{2\times 60\mu\times 5\times 150\,\mu} = 300\mu\mho$$

$$A_v = -g_m\left(r_d \parallel r_d\right) = \left(-300\frac{\mu A}{V}\right)\left(6M\Omega \| 16M\Omega\right)$$

$$= \left(-300\frac{\mu A}{V}\right)\left(3M\Omega\right) = -900\,V/V$$

27. It is desired to find three-tap causal filter which gives zero signal as an output to and input of the form

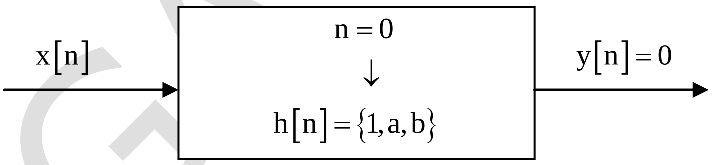
$$x[n] = c_1\exp\left(-\frac{j\pi n}{2}\right) + c_2\exp\left(\frac{j\pi n}{2}\right),$$

Where c_1 and c_2 are arbitrary real numbers. The desired three-tap filter is given by

$h[0]=1, \quad h[1]=a, \qquad h[2]=b$ and

$h[n]=0$ for $n<0$ or $n>2$.

What are the values of the filter taps a and b if the output is $y[n]=0$ for all n, when x[n] is as given above?



(A) $a=-1, b=1$

(B) $a=0, b=1$

(C) $a=1, b=1$

(D) $a=0, b=-1$

Key: (B)

Sol: It is given that

$h(n) = [1, a, b]$

$x(n) = C_1 e^{-j\frac{\pi}{2}n} + C_2 e^{j\frac{\pi}{2}n}$

$\Rightarrow y(n) = 0$

$\rightarrow$ If $h(n) = [1, a, b]$

$H(e^{j\omega}) = 1 + ae^{-j\omega} + be^{-j2\omega}$

$\rightarrow$ when $x(n) = C_1 e^{-j\frac{\pi}{2}n} + C_2\, e^{j\frac{\pi}{2}n}$ then
exiression of $y(n)$

Since the input x (n) contain 2 frequencies $\pm\frac{\pi}{2}$,

Let evaluate $\left|H\left(e^{j\omega}\right)\right|$ at f nil 2 frequency

$$H\left(e^{j-\pi/2}\right) = 1 + ae^{-j\left(\frac{-\pi}{2}\right)} + be^{-j2\left(\frac{-\pi}{2}\right)}$$
$$= 1 + ae^{+j\pi/2} + be^{j\pi}$$
$$= 1 + \left[a(j)\right] + \left[b(-1)\right]$$
$$= (1-b) + j(a)$$
$$H\left(e^{j\pi/2}\right) = (1-b) - ja$$
$$\left|H\left(e^{j\pi/2}\right)\right| = \left|H\left(e^{-j\frac{\pi}{2}}\right)\right| = \sqrt{(1-b)^2 + a^2}$$

So the expression of y(n) is

$$y(n) = \left[(1-b)^2 + a^2\right]^{1/2} C_1 e^{-j\left(\frac{1}{2}n+\phi_1\right)} + \left[(1-b)^2 + a^2\right]^{1/2} C_2 e^{j\left(\frac{\pi}{2}n+\phi_2\right)}$$

if we want $y(n) = 0$, it means

$k = \sqrt{(1-b)^2 + a^2} = 0$ (from the above expression)

If we check each options

(A) $a = -1, b = 1$, then $k = \sqrt{0^2 + 1^2} = 1$ (not correct)

(B) $a = 0, b = 1$ then $k = \sqrt{0^2 + 0^2} = 0$ (correct)

(C) $a = 1, b = 1$, then $k = \sqrt{0^2 + 1^2} = 1$ (not correct)

(D) $a = 0, b = 1$, then $k = \sqrt{2^2 + 0^2} = 2$ (not correct)

So a = 0, b = 1, is correct.

28. Let h[n] be length-7 discrete-time finite impulse response filter, given by

$$h[0] = 4,\ h[1] = 3,\ h[2] = 2,\ h[3] = 1$$
$$h[-1] = -3,\ h[-2] = -2,\ h[-3] = -1,$$

and $h[n]$ is zero for $|n| \geq 4$. A length-3 finite impulse response approximation g[n] of h[n] has to be obtained such that

$$E(h,g) = \int_{-\pi}^{\pi} \left|H\left(e^{j\omega}\right) - G\left(e^{j\omega}\right)\right|^2 d\omega$$

is minimized, where $H\left(e^{j\omega}\right)$ and $G\left(e^{j\omega}\right)$ are the discrete-time Fourier transforms of h[n] and g[n], respectively. For the filter that minimizes E(h,g), the value of $10g[-1]+g[1]$, rounded off to 2 decimal places, is_______.

Key: (–27)

Sol: It is given that

$$h(n)=[-1,-2,-3,4,3,2,1] \quad ....(1)$$

And it is also mentioned that g(n) is derived from h(n) having 3 samples, from the information given in question we need g(1), and g(-1) so left

$$g(n)=\left[a,\underset{\uparrow}{b},c\right] \quad ...(2)$$

It is mentioned that

$$E(h,g)]\int_{-x}^{\pi}\left|H\left(e^{j\omega}\right)-G\left(e^{jw}\right)\right|^2 d\omega, \text{ is minimised,}$$

If $h(n)$ and $g(n)$ represent IDTFT of $H\left(e^{j\omega}\right), G\left(e^{j\omega}\right)$

then $E(h,g)=2\pi\Sigma|h(n)-g(n)|^2$ (by Parseval theorem)

$$h(n)\leftrightarrow H\left(e^{j\omega}\right)$$

$$g(n)\leftrightarrow G\left(e^{j\omega}\right)$$

$$h(n)-g(n)\leftrightarrow H\left(e^{j\omega}\right)-G\left(e^{j\omega}\right)$$

$$\text{energy of }[h(n)-g(n)]\leftrightarrow\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H\left(e^{j\omega}\right)-G\left(e^{j\omega}\right)\right|^2 d\omega$$

$$\Rightarrow 2\pi\text{ energy of }[h(n)-g(n)]=\int_{-\pi}^{\pi}\left|H\left(e^{j\omega}\right)-G\left(e^{j\omega}\right)\right|^2 d\omega$$

$$\Rightarrow E(h,g)=\int_{-x}^{\pi}\left|H\left(e^{j\omega}\right)-G\left(e^{j\omega}\right)\right|^2 d\omega=2\pi\sum_{n=\infty}^{\infty}|h(n)-g(n)|^2$$

We want to minimize E (h,g)

Using equation (1) and equation (2) we can say

$$h(n)-g(n)=[-1,-2,-3-a,4-b,3-c,2,1]$$

$$\rightarrow E(h,g)=2\pi\Sigma|h(n)-g(n)|^2=2\pi\Sigma[h(n)-g(n)]^2$$

$[\because$ all elements are real, mod does not have significance$]$

$$\rightarrow E(h,g)=2\pi\left[(-1)^2+(-2)^2+(-3-a)^2+(4-b)^2+(3-c)^2+2^2+1^2\right]$$

$$=2\pi\left[10+(-3-a)^2+(4-b)^2+(3-1)^2\right]$$

To have minimum E (h,g), we need

$-3-a=0 \Rightarrow a=-3$

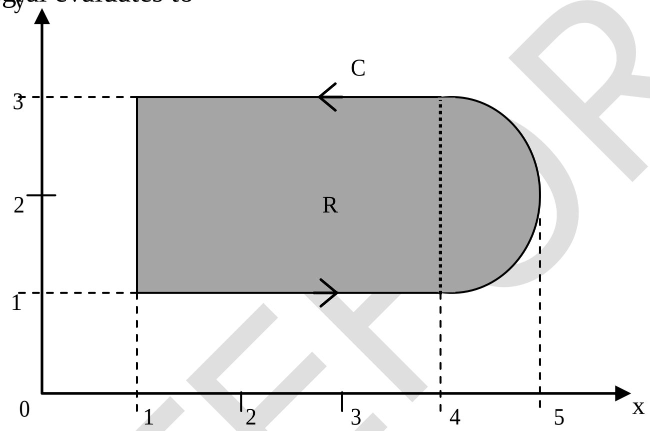
$4-b=0 \Rightarrow b=4$

$3-c=0 \Rightarrow c=3$

$\rightarrow g(n) = \left[a, \underset{\uparrow}{b}, c\right] = \left[-3, \underset{\uparrow}{4}, 3\right]$

So $10g(-1)+g(1)=10a+c=\left[10(-3)\right]+3=-30+3=-27$

29. Consider the line integral $\int_C (xdy - ydx)$. The integral being taken in a counterclockwise direction over the closed curve C that forms the boundary of the region R shown in the figure below. The region R is the area enclosed by the union of a 2×3 rectangle and a semi-circle of radius 1. The line integral evaluates to



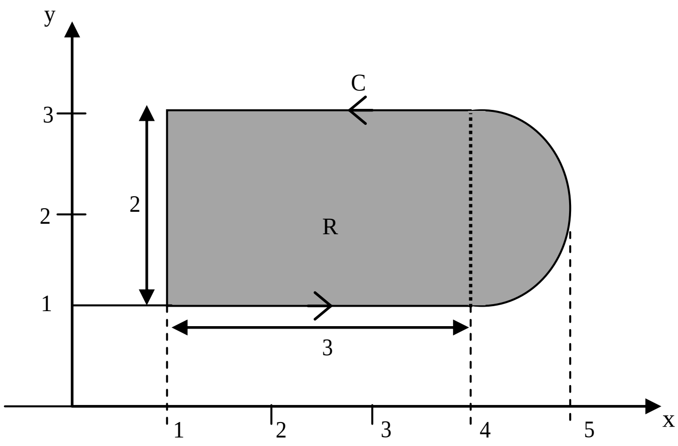
(A) $16+2\pi$ (B) $6+\pi/2$ (C) $12+\pi$ (D) $8+\pi$

Key: (C)

Sol: $\int_C (x\,dy - ydx) = \int_C (-y\,dx + x\,dy)$; where C is a

Closed figure formed by rectangle and a semi circle of radius 1.

Using Green's theorem, we have



$\oint_C \left(-\underset{M}{\underset{\downarrow}{y}}\, dx + \underset{N}{\underset{\downarrow}{x}}\, dy\right) = \iint_R (1+1)\,dx\,dy$

$= 2\iint_R dx\,dy \rightarrow$ Area of the region R.

$= 2\left[\text{Area of rectangle} + \text{Arae of semi-circle}\right]$

$= 2\left[(3\times2)+\frac{1}{2}\pi\left(1^2\right)\right]$

$= 2\left[6+\pi/2\right] = (12+\pi)$

30. A rectangular waveguide of width w and height h has cut-off frequencies for TE_{10} and $T_{E_{11}}$ modes in the ratio 1:2. The aspect ratio w/h, rounded off to two decimal places, is_______.

Key: (1.732)

Sol: Cut off frequency of TE_{10} mode is $f_{C_{10}} = \dfrac{C}{2W}$

Cutoff frequency of TE_{11} mode is $f_{C_{11}} = \dfrac{C}{2W}\sqrt{1^2 + \left(\dfrac{W}{h}\right)^2}$

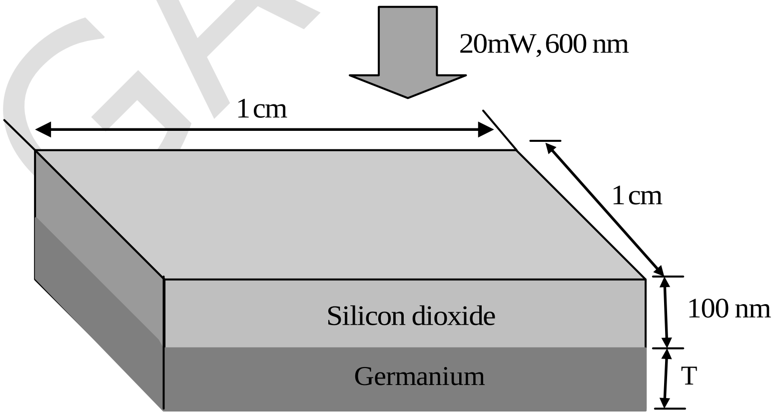
Given $\dfrac{f_{C_{10}}}{f_{C_{11}}} = \dfrac{1}{2} \Rightarrow \dfrac{(C/2W)}{\left(\dfrac{C}{2W}\right)\sqrt{1^2 + \left(\dfrac{W}{h}\right)^2}} = \dfrac{1}{2}$

$\Rightarrow 1^2 + \left(\dfrac{W}{h}\right) = 4$

$\Rightarrow \dfrac{W}{h} = \sqrt{3}$

$\therefore$ Aspect ratio= $\dfrac{W}{h} = \sqrt{3} = 1.732.$

31. A Germanium sample of dimensions $1\text{cm} \times 1\text{cm}$ is illuminated with a 20 mW, 600 nm laser light source as shown in the figure. The illuminated sample surface has a 100 nm of loss-less Silicon dioxide layer that reflects one-fourth of the incident light. From the remaining light, one-third of the power is reflected form the silicon dioxide- Germanium interface, one-third is absorbed in the Germanium layer, and one-third is transmitted through the other side of the sample. If the absorption coefficient of Germanium at 600 nm is $3\times10^4\ \text{cm}^{-1}$ and the bandgap is 0.66 eV, the thickness of the Germanium layer, rounded off to 3 decimal places, is _____ μm.



Key: (0.231)

Sol: $0 \leftarrow P(0) = 10mW$

$T \leftarrow P(T) = 5mW$

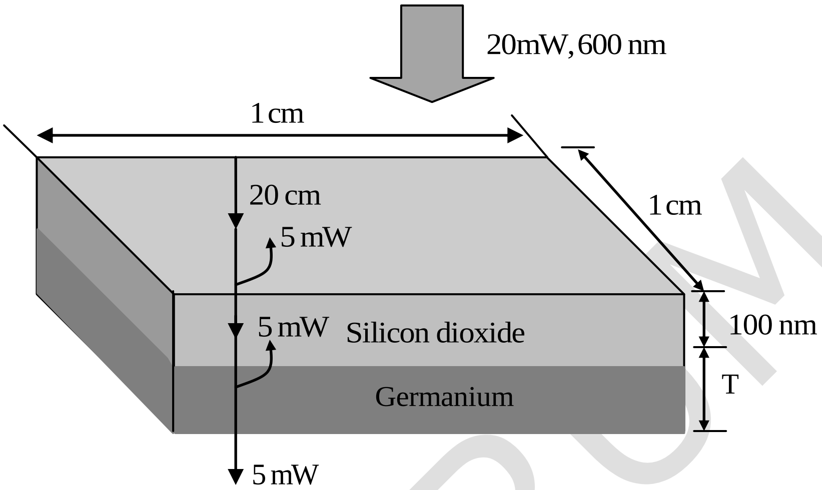
$\alpha = 3\times10^4 cm^{-1} = 3\times10^6 m^{-1}$

$P(T) = P(0)e^{-\alpha T}$

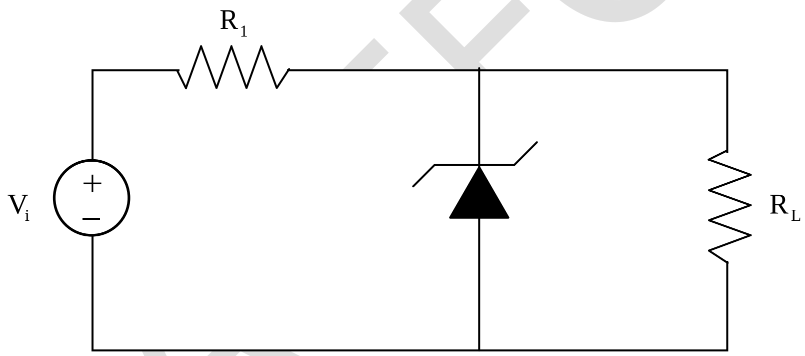
$$T = \frac{1}{\alpha}\ell n\left[\frac{P(0)}{P(T)}\right]$$

$$T = \frac{1}{3\times10^6}\ell n\left(\frac{10}{5}\right)$$

$= 0.23\times10^{-6} m = 0.231\,\mu m$



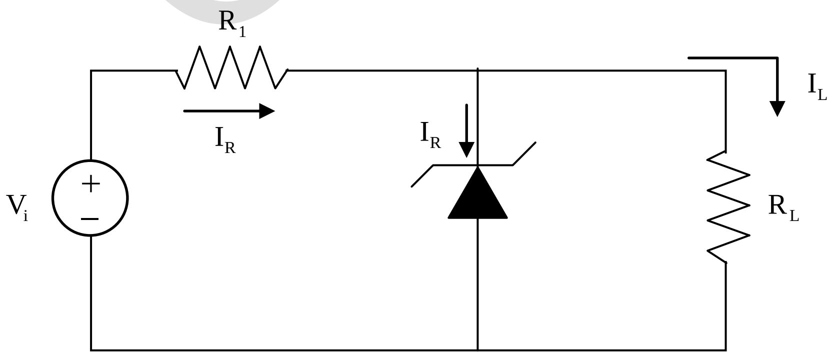
32. In the circuit shown, the breakdown voltage and the maximum current of the Zener diode are 20 V and 60 mA, respectively. The values of R_1 and R_L are $200\,\Omega$ and $1\,k\Omega$, respectively. What is the range of V_i that will maintain the Zener diode in the 'n' state?



(A) 24 V to 36V (B) 22 V to 34 V (C) 20 V to 28 V (D) 18 V to 24 V

Key: (A)

Sol:



$$I_L = \frac{20-0}{1} mA = 20mA$$

$I_R = I_Z + I_L$

$V_{i\,min} = I_{R\,min} R_1 + V_Z$

$I_{R\,min} = 0 + I_2 \left(\because I_{z(min)} = 0 \right) = 20mA.$

$V_{i\,min} = 20 \times 10^{-3} \times 100 + 2 = 4V + 20v = 24v$

$V_{i\,max} = I_{R\,max} R_1 + V_z$

$I_{R\,max} = I_{z\,max} + I_L = 60\ mA + 20mA = 80mA.$

$V_{i\,max} = 80 \times 10^{-3} \times 200 + 20 = 16V + 20V = 36V$

$\therefore V_{i\,min} = 24V; \quad V_{i\,max} = 36V$

33. A single bit, equally likely to be 0 and 1, is to be sent across an additive white Gaussian noise (AWGN) channel with power spectral density $N_0/2$. Binary signaling, with $0 \to p(t)$ and $1 \to q(t)$, is used for the transmission, along with an optimal receiver that minimizes the bit-error probability.

Let $\varphi_1(t), \varphi_2(t)$ form and orthonormal signal set.

If we choose $p(t) = \varphi_1(t)$ and $q(t) = -\varphi_1(t)$, we would obtain a certain bit-error probability P_b.

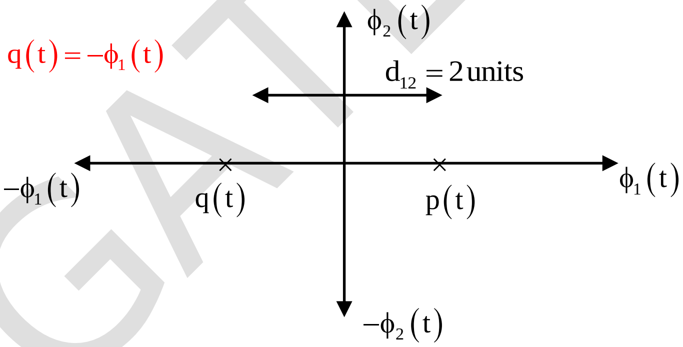
If we keep $p(t) = \varphi_1(t)$, but take $q(t) = \sqrt{E}\varphi_2(t)$, for what value of E would we obtain the same bit-error probability P_b?

(A) 3 (B) 1 (C) 2 (D) 0

Key: (A)

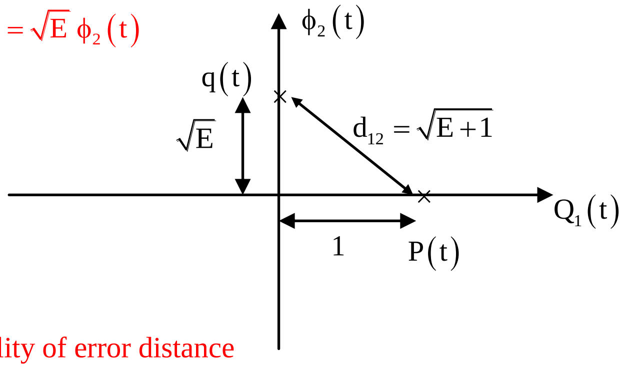
Sol: Case 1:

$P(t) = \phi_1(t), q(t) = -\phi_1(t)$



Case 2:

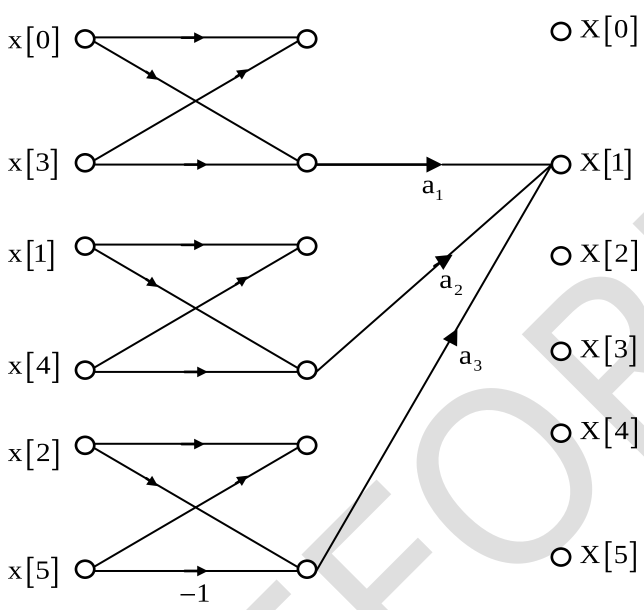
$P(t) = \phi_1(t) \;\; q(t) = \sqrt{E}\ \phi_2(t)$



For same probability of error distance between points should be same for both cases

$\therefore \sqrt{E+1} = 2 \;\; \therefore E = 3$

34. Consider a six-point decimation-in-time Fast Fourier Transform (FFT) algorithm, for which the signal-flow graph corresponding to X[1] is shown in the figure. Let $W_6 = \exp\left(-\frac{j2\pi}{6}\right)$. In the figure, what should be the values of the coefficients a_1, a_2, a_3 in terms of W_6 so that X[1] is obtained correctly ?



(A) $a_1 = 1, a_2 = W_6^2, a_3 = W_6$ (B) $a_1 = -1, a_2 = W_6^2, a_3 = W_6$

(C) $a_1 = -1, a_2 = W_6, a_3 = W_6^2$ (D) $a_1 = 1, a_2 = W_6, a_3 = W_6^2$

Key: (D)

Sol: In this case we are supposed to obtain the FFT coefficient $[X(1)]$ using DIT algorithm.

We are supposed to obtain the coefficient a_1, a_2, a_3

The given butterfly structure is a standard structure where

$a_1 = W_6^0 = 1$

$a_2 = W_6^1 = W_6$

$a_3 = W_6^2$

35. The quantum efficiency (η) and responsivity (R) at a wavelength λ (in μm) in a p-i-n photo detector are related by

(A) $R = \frac{\eta \times \lambda}{1.24}$ (B) $R = \frac{\lambda}{\eta \times 1.24}$ (C) $R = \frac{1.24 \times \lambda}{\eta}$ (D) $R = \frac{1.24}{\eta \times \lambda}$

Key: (A)

Sol: Responsivity $(R) = \frac{e\eta}{hv} = \frac{e\eta s}{hc} = \frac{\eta \times \lambda}{1.24}(A/w)$

36. Consider a long-channel MOSFET with a channel length $1\mu m$ and width $10\mu m$. The device parameters are acceptor concentration $N_A = 5\times10^{16}\ cm^{-3}$, electron mobility $\mu_n = 800\ cm^2/V-s$, oxide capacitance/area $C_{ox} = 3.45\times10^{-7}\ F/cm^2$, threshold voltage $V_T = 0.7V$. The drain saturation current (I_{Dsat}) for a gate voltage of 5V is _______mA (rounded off to two decimal places). $\left[\varepsilon_0 = 8.854\times10^{-14}\ F/cm, \varepsilon_{Si} = 11.9\right]$

Key: (25.51)

Sol: Given data; $L = 1\mu m, W = 10\mu m$

$$N_A = 5\times10^{16}\,cm^{-3}, \mu_n = 800\frac{cm^2}{v-sec}$$

$$C_{ox} = 3.45\times10^{-7}\ F/cm, V_T = 0.7V$$

$$\epsilon_0 = 8.854\times10^{-14}\ F/cm, \epsilon_{si} = 11.9$$

$$V_G = 5V$$

$$I_{Dsat} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}[V_G - V_T]^2$$

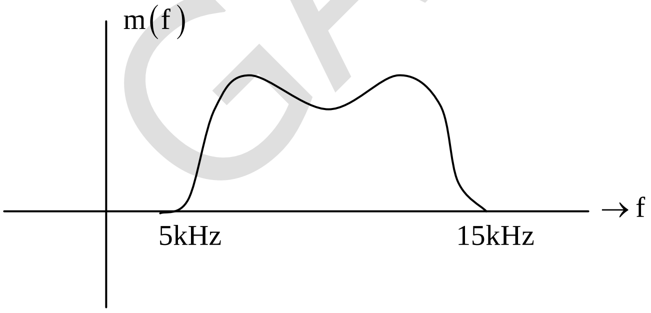
$$= \frac{1}{2}\times800\times3.45\times10^{-7}\times\frac{10}{1}[5-0.7]^2 = 0.0255A = 25.5162\ mA \approx 25.51mA$$

37. A voice signal m (t) is in the frequency range 5 kHz to 15 kHz. The signal is amplitude modulated to generate an AM signal $f(t) = A(1+m(t))\cos 2\pi f_c t$, where $f_c = 600$ kHz.

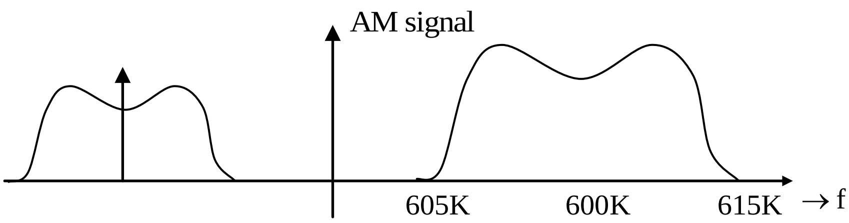
The Am signal f(t) is to be digitized and archived. This is done by first sampling f(t) at 1.2 times the Nyquist frequency, and then quantizing each sample using a 256-level quantizer. Finally, each quantized sample is binary coded using K bits, where K is the minimum number of bits required for the encoding. The rate, in Megabits per second (rounded off to 2 decimal places), of the resulting stream of coded bits is _______Mbps.

Key: (0.192)

Sol:



Modulation done using AM



Now AM signal id done sampling

$$\frac{2f_{+1}}{n} \le f_s \le \frac{2f_1}{n-1}$$

Here $f_H = 615K, f_L = 605\,K.$

$$1 \le n \le \frac{f_H}{B} : B = f_H - f_L$$

$n \le 61.5, n = 61.$

$$f_s \ge \frac{2\times 615}{61}$$

Minimum sampling is 20 K.

$R_b = 1.2 f_s \times 8 = 0.192M$ bit/s

38. A random variable X takes values -1 and +1 with probabilities 0.2 and 0.8, respectively. It is transmitted across a channel which adds noise N, so that the random variable at the channel output is Y = X +N. The noise N is independent of X, and is uniformly distributed over the interval [-2, 2]. The receiver makes a decision

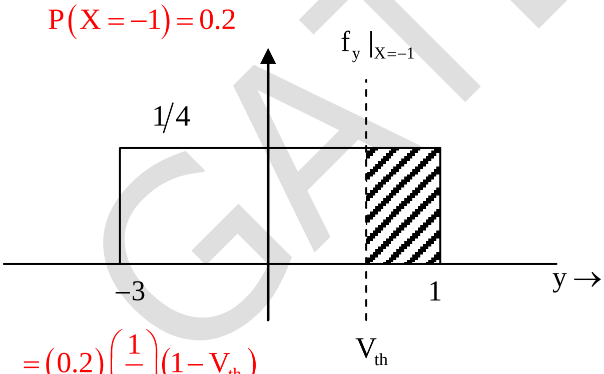
$$X = \begin{cases} -1, \text{ if } Y \le \theta \\ +1, \text{ if } Y > \theta \end{cases}$$

Where the threshold $\theta \in [-1,1]$ is chosen so as to minimize the probability of error $\Pr[X \ne X]$.

The minimum probability of error, rounded off to 1 decimal place, is _______.

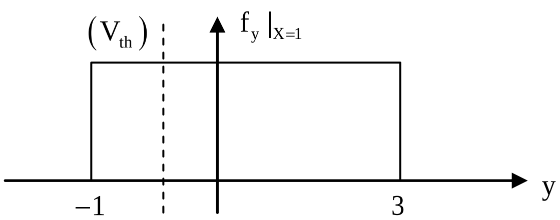
Key: (0.1)

Sol: $\frac{\text{When } X = -1 \text{ is transmitted}}{P(X=-1)=0.2}$



$$P_{e_{-1}} = (0.2)\left(\frac{1}{4}\right)(1 - V_{th})$$

When $X = 1$ is trasmitted $P[X = 1] = 0.8$



$$P_{e_1} = (0.8)\left(\frac{1}{4}\right)(V_{th} + 1)$$

$$P_e = \frac{(0.2)(1)(1 - V_{th}) + (0.8)(1)(V_{th} + 1)}{4}$$

$$P_e = \frac{1 + 0.6V_{th}}{4}$$

$$-1 \le V_{th} \le 1$$

$$\therefore P_{e_{min}} \text{ when } V_{th} = -1$$

$$\therefore P_{e_{min}} = \frac{1 + (0.6)(-1)}{4} = \frac{0.4}{4} = 0.1$$

39. Let the state-space representation of an LTI system be $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + d\,u(t)$ where A,B,C are matrices, d is a scalar, u(t) is the input to the system, and y(t) is its output. Let $B = [0\ \ 0\ \ 1]^T$ and $d = 0$. Which one of the following options for A and C will ensure that the transfer function of this LTI system is $H(s) = \dfrac{1}{s^3 + 3s^2 + 2s + 1}$?

(A) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$ and $C = [1\ \ 0\ \ 0]$

(B) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$ and $C = [0\ \ 0\ \ 1]$

(C) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$ and $C = [0\ \ 0\ \ 1]$

(D) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$ and $C = [1\ \ 0\ \ 0]$

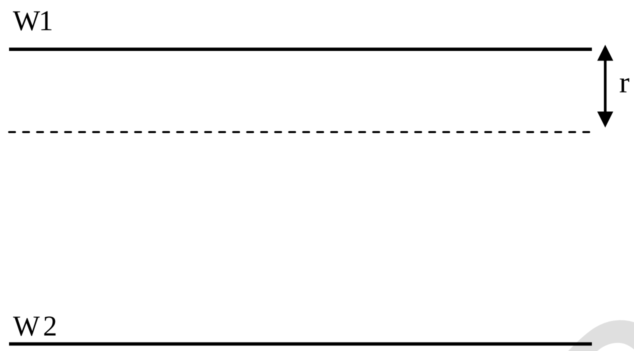
Key: (A)

Exp: $H(s) = \dfrac{1}{s^3 + 3s^2 + 2s + 1}$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}}_{(\text{A matrix})} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \underbrace{[1\ \ 0\ \ 0]}_{(\text{C matrix})} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

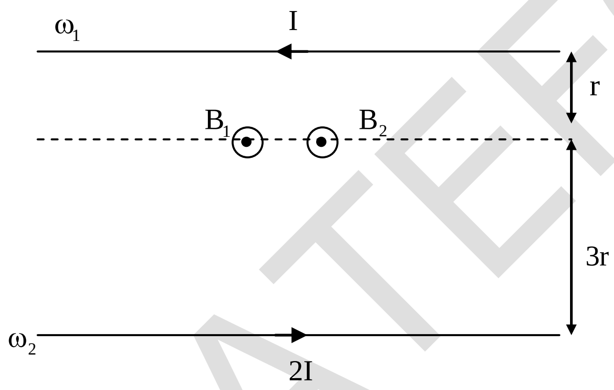
40. Two identical copper wires W1 and W2, placed in parallel as shown in the figure, carry currents I and 2I, respectively, in opposite directions. If the two wires are separated by a distance of 4r, then the magnitude of the magnetic field $\vec{B}$ between the wires at a distance r form W1 is



(A) $\frac{5\mu_0 I}{6\pi r}$ (B) $\frac{\mu_0 I}{6\pi r}$ (C) $\frac{6\mu_0 I}{5\pi r}$ (D) $\frac{\mu_0^2 I^2}{2\pi r^2}$

Key: (A)

Sol:



Between the wires ω_1 and ω_2 the $\overline{B}$ – fields due to I and 2I will gets added up.

$\left.\begin{array}{l}\therefore \text{The B – field due to } \omega_1 \\ \text{at a distance 'r' from from} \\ \text{the } \omega_1 \text{ is}\end{array}\right\} B_1 = \frac{\mu_0 I}{2\pi r} \rightarrow (1)$

$\left.\begin{array}{l}\text{The B – field due to } \omega_2 \\ \text{at a distance "3r" form} \\ \text{the } \omega_2 \text{ as shown in} \\ \text{figure is}\end{array}\right\} B_2 = \frac{\mu_0 (2I)}{2\pi(3r)} \rightarrow (2)$

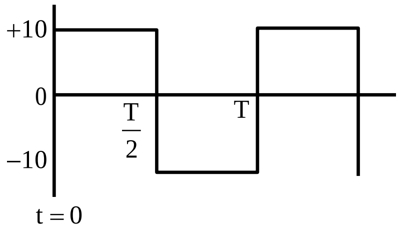
$\therefore$ The total B – field $B = B_1 + B_2$

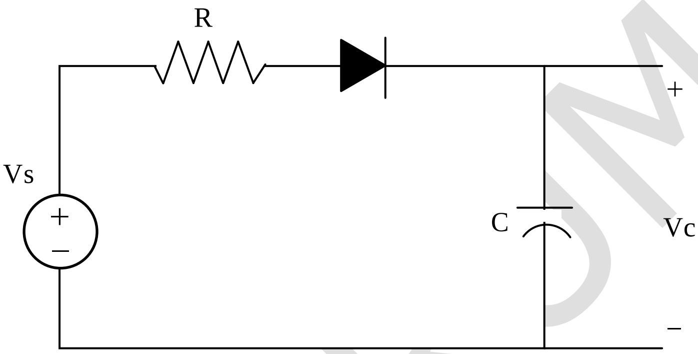
$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{r} + \frac{2}{3r}\right]$$

$$B = \frac{5\mu_0 I}{6\pi r}\,\omega b/m^2.$$

41. In the circuit shown, V_S is a 10V square wave of period, T=4 ms with $R = 500\Omega$ and $C = 10\,\mu F$. The capacitor is initially uncharged at t = 0, and the diode is assumed to be ideal.

The voltage across the capacitor (V_c) at 3 ms is equal to ______volts (rounded off to one decimal place).



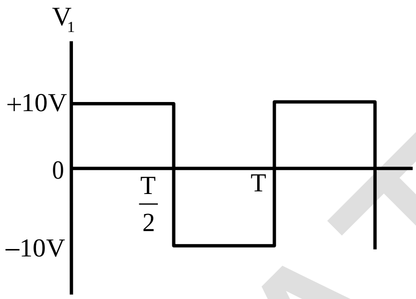


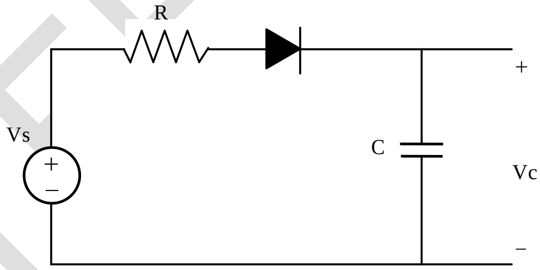
Key: (3.31)

Sol: Given: $T = 4ms,\ R = 500\Omega,\ C = 10\mu F$

$$\tau = RC = 500 \times 10 \times 10^{-6}\ sec = 5ms$$

$$\frac{T}{2} = \frac{4ms}{2} = 2ms$$





For positive half cycle, diode will be forward biased and capacitor start to charge

$$V_C = V_C(\infty)\left[1 - e^{\frac{-t}{RC}}\right] = 10\left[1 - e^{-\frac{2ms}{5ms}}\right]$$

$$= 10\left[1 - e^{-\frac{2}{5}}\right] = 4.51V \Rightarrow \boxed{\therefore V_C = 3.31V}$$

42. Consider a causal second-order system with the transfer function $G(s) = \frac{1}{1 + 2s + s^2}$

With a unit-step $R(s) = \frac{1}{s}$ as an input. Let C(s) be the corresponding output. The time taken by the system output C(t) to reach 94% of its steady-state value $\lim_{t \to \infty} c(t)$, rounded off to two decimal places, is

(A) 5.25 (B) 2.81 (C) 4.50 (D) 3.89

Key: (C)

Sol: Given, $G(s)=\dfrac{1}{1+2s+s^2}$

$$\therefore C(s)=G(s)\,R(s)=\frac{1}{s(s+1)^2}$$

$$C(t)=1-e^{-t}-te^{-t}$$

Let's check option one by one

Option (A)

$$0.94=1-e^{-5.25}-5.25\,e^{-5.25}=0.89$$

(Hence option A is wrong)

Option (C)

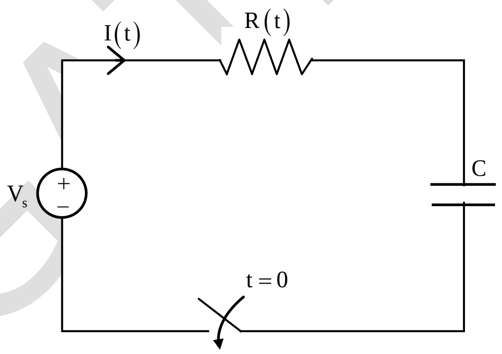
$$0.94=1-e^{-4.50}-4.50e^{-4.50}=0.94$$

(Hence option (C) is correct)

43. The RC circuit shown below has a variable resistance R(t) given by the following expression:

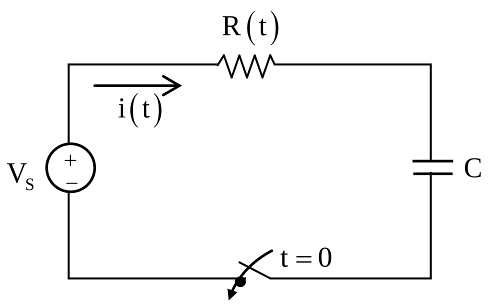
$$R(t)=R_0\left(t-\frac{t}{T}\right)\text{for } 0\le t<T$$

Where $R_0=1\Omega, C=1F$. We are also given that $T=3R_0C$ and the source voltage is $V_S=1V$. If the current at time t = 0 is 1A, then the current I(t), in amperes, at time t = T/2 is_________(rounded off to 2 decimal places).



Key: (0.25)

Exp: The given circuit is as shown in figure



Where $R(t) = R_0\left(1-\frac{t}{T}\right)$ for $0 \le t \le T$

and $(R_0 = 1\Omega)(C = 1F), (T = 3R_0C), (V_s = 1V)(i(0) = 1A)$

We heed to obtain i(t) a $t = \frac{T}{2}$

$$\begin{bmatrix} T = 3\,(\because R_0 = 1\Omega, C = 1F) \\ R(t) = \left(1-\frac{t}{3}\right) \\ \frac{T}{2} = 1.5 \end{bmatrix}$$

By KVL we have

$$i(t)R(t) + \frac{1}{C}\int i\,dt = V_s$$

$$\Rightarrow R(t)\frac{di(t)}{dt} + i(t)\frac{dR(t)}{dt} + \frac{1}{C}i = 0$$

$$\Rightarrow \left(1-\frac{t}{3}\right)\frac{di(t)}{dt} + i(t)\left(\frac{-1}{3}\right) + i = 0$$

$$\Rightarrow (3-t)\frac{di}{dt} = -2i$$

$$\Rightarrow \frac{di}{i} = \frac{-2}{3-t}dt$$

$$\Rightarrow \ell n i = 2\ell n(t-3) + \ell n c$$

$$\Rightarrow \ell n i = \ell n\left[(t-3)^2 .C\right]$$

$$\Rightarrow i(t) = (t-3)^2 C$$

$$\Rightarrow i(0) = (0-3)^2 C$$

$$\Rightarrow C = \frac{1}{9}(\because i(0) = 1, \text{given})$$

$$i(t) = (t-3)^2\frac{1}{9}$$

$$i(1.5) = \frac{1}{9}(-1.5)^2 = 0.25A$$

44. In an ideal p-n junction with an ideality factor of 1 at T=300 K, the magnitude of the reverse-bias voltage required to reach 75% of its reverse saturation current, rounded off to 2 decimal places, is ______mV.

$$\left[k = 1.38\times10^{-23}\,JK^{-1},\ h = 6.625\times10^{-34}\,J-s,\ q = 1.602\times10^{-19}\,C\right]$$

Key: (35.87)

Sol: $I = I_o\left[\exp\left(\frac{-V_R}{V_T}\right)-1\right] = \frac{-3}{4}I_o = I_o\left[\exp\left(\frac{-V_R}{V_T}\right)-1\right]$

$$\exp\left[\frac{-V_R}{V_T}\right] = \frac{-3}{4}+1 = \frac{1}{4}$$

$$-V_R = V_T\,\ell n\left(\frac{1}{4}\right) = 25.9\times\ell n\left(\frac{1}{4}\right)mV$$

$$|-V_R| = 35.87mV$$

45. The dispersion equation of a waveguide, which relates the wave number k to the frequency ω, is

$$k(\omega) = (1/c)\sqrt{\omega^2-\omega_0^2}$$

Where the speed of light $c = 3\times10^8\,m/s$, and ω_0 is a constant. If the group velocity is $2\times10^8\,m/s$, then the phase velocity is

(A) $2\times10^8\,m/s$ (B) $1.5\times10^8\,m/s$ (C) $3\times10^8\,m/s$ (D) $4.5\times10^8\,m/s$

Key: (D)

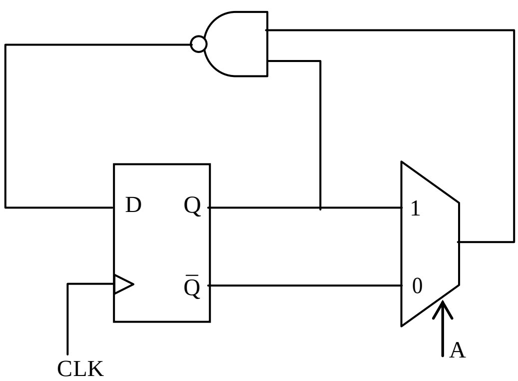
Sol: Given, $C = 3\times10^8\,m/sec$

Group velocity $v_g = 2\times10^8\,m/sec$

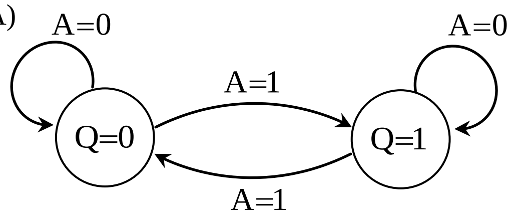
$v_p v_g = C^2$

$\therefore$ Phase velocity $v_p = \frac{C^2}{v_g} = \frac{(3\times10^8)^2}{2\times10^8} = 4.5\times10^8\,m/sec$

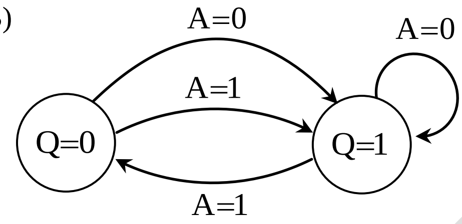
46. The state transition diagram for the circuit shown is



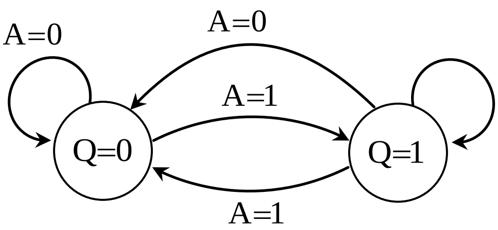
(A)



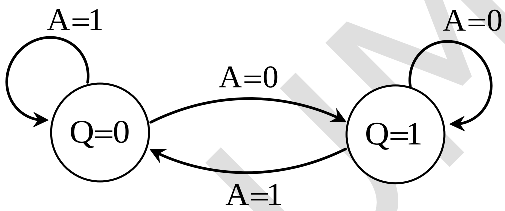
(B)



(C)

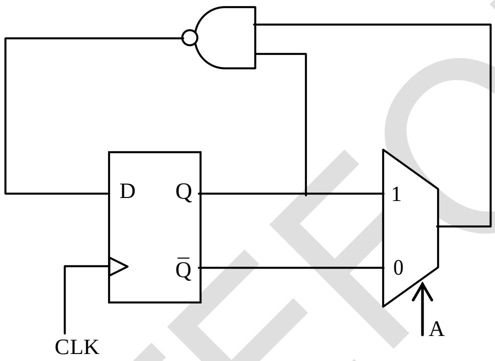


(D)



Key: (B)

Sol: The given circuit is



Let $Q = 0$

$\rightarrow$ When $Q = 0, \bar{Q} = 1$ and $A = 0$,

then $Y = \bar{Q} = 1$, so $D = \overline{QY} = \overline{0.1} = \bar{0} = 1$

after 1 clock $Q^{+} = 1$

$\rightarrow$ When $Q = 0, \bar{Q} = 1$ and $A = 1$,

then $Y = Q = 1$, so $D = \overline{QY} = \overline{0.0} = 1$

after 1 clock $Q^{+} = 1$

Let $Q = 1$

$\rightarrow$ When $Q = 1, \bar{Q} = 0$ and $A = 0$,

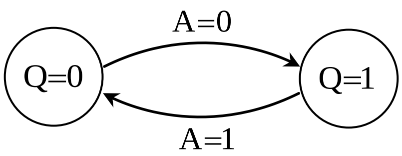
then $Y = \bar{Q} = 0$, so $D = \overline{QY} = \overline{1.0} = \bar{0} = 1$

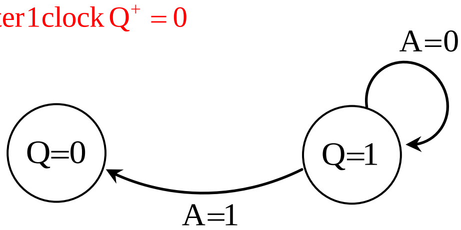
after 1 clock $Q^{+} = 1$

$\rightarrow$ When $Q = 1, \bar{Q} = 0$ and $A = 1$,

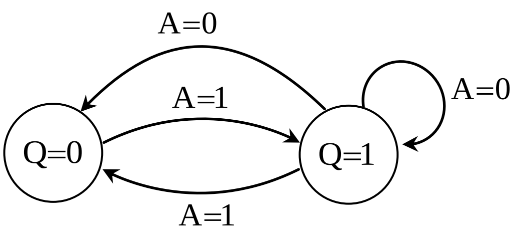
then $Y = Q = 1$, so $D = \overline{QY} = \overline{1.1} = 0$

after 1 clock $Q^{+} = 0$





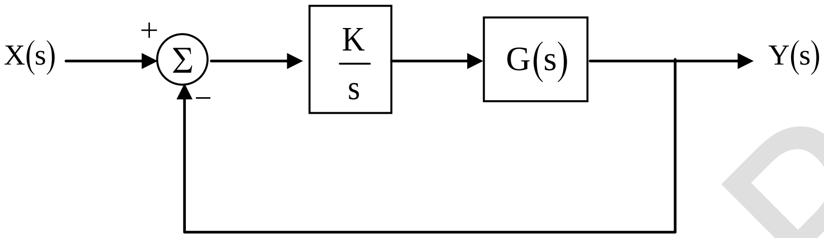
By combining both we can draw a single state diagram



47. Consider a unity feedback system, as in the figure shown, with an integral compensator $\frac{k}{s}$ and open-loop transfer function

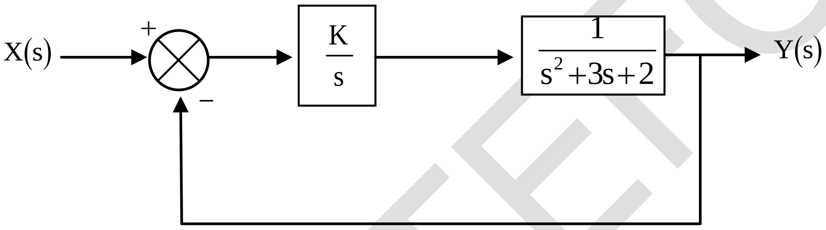
$$G(s)=\frac{1}{s^2+3s+2}$$

Where K>0. The positive value of K for which there are exactly two poles of the unity feedback system on the $j\omega$ axis is equal to ______ (rounded off to two decimal places).



Key: (6)

Sol: Given, A unity feedback system as shown in figure



$$CE = s\left(s^2+3s+2\right)+k=0$$

$$\Rightarrow s^3+3s^2+2+k=0$$

s^3	1	2
s^2	3	k
s^1	$\boxed{\frac{6-k}{3} \quad 0}$	
s^0	k	0

If this row will be zero then two poles will be on $j\omega$ axis

$$6-K=0 \Rightarrow \boxed{K=6}$$

Lets cross check : $3s^2+6=0 \Rightarrow s^2+2=0$

$\boxed{s=\pm j2}$ → Two poles on $j\omega$

48. Consider a differentiable function $f(x)$ on the set of real numbers such that $f(-1)=0$ and $|f'(x)| \le 2$. Given these conditions, which one of the following inequalities is necessarily true for all $x \in [-2,2]$?

(A) $f(x) \le 2|x+1|$ (B) $f(x) \le 2|x|$ (C) $f(x) \le \frac{1}{2}|x+1|$ (D) $f(x) \le \frac{1}{2}|x|$

Key: (A)

Sol: Method-I:

Option (A) satisfy the given conditions $\left(f(-1)=0 \ \& \ |f'(x)| \le 2\right)$

Method-II:

Given that $|f'(x)| \le 2;\ f(-1)=0$

$\Rightarrow -2 \le f'(x) \le 2;$ where $x \in [-2, 2]$

Using Lagrange's Mean Value Theorem over $[-1, 2]$, we have

$$\Rightarrow -2 \le f'(x) \le 2 \Rightarrow -2 \le \frac{f(2)-f(-1)}{2-(-1)} \le 2 \quad \left[\because f'(c) = \frac{f(b)-f(a)}{b-a}\right]$$

$$\Rightarrow -2 \le \frac{f(2)-0}{3} \le 2$$

$$\Rightarrow -6 \le f(2) \le 6 \qquad ...(2)$$

$\therefore$ Option (A) satisfies equation (2).

49. Consider the homogeneous ordinary differential equation $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0,\ x>0$

With $y(x)$ as a general solution. Given that $y(1)=1$ and $y(2)=14$ the value of $y(1.5)$, (rounded off to two decimal places), is _________.

Key: (5.25)

Sol: Given D.E

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0, \qquad ...(1)$$

$$x>0 \ \& \ y(1)=1 \text{ and } y(2)=14$$

Clearly equation(1) is Cauchy-Euler Linear Differential Equation.

Equation (1) can be written as

$$\left[x^2D^2 - 3xD + 3\right]y = 0$$

Let $xD = \theta;\ x^2 D^2 = \theta(\theta-1);$ where $\theta = \frac{d}{dz}$ and $x = e^z$

$\therefore \left[\theta(\theta-1) - 3\theta + 3\right] y = 0 \ \Rightarrow \left[\theta^2 - 4\theta + 3\right] y = 0$

Consider the A.E is $\theta^2 - 4\theta + 3 = 0 \Rightarrow (\theta-3)(\theta-1) = 0$

$\Rightarrow \theta = 1, 3 \rightarrow$ Roots are real & distinct

$\therefore$ The solution is, $y = C_1 e^{1.z} + C_2 e^{3.z} \Rightarrow y = C_1 x + C_2 x^3$ ...(2)

$\left[\because x = e^z\right]$

Given, $y = 1$ at $x = 1 \Rightarrow 1 = C_1 + C_2\ (\because (2))$ ...(3)

& $y = 14$ at $x = 2 \Rightarrow 14 = 2C_1 + 8C_2\ \left[\because (2)\right]$

$\Rightarrow C_1 + 4C_2 = 7$ ...(4)

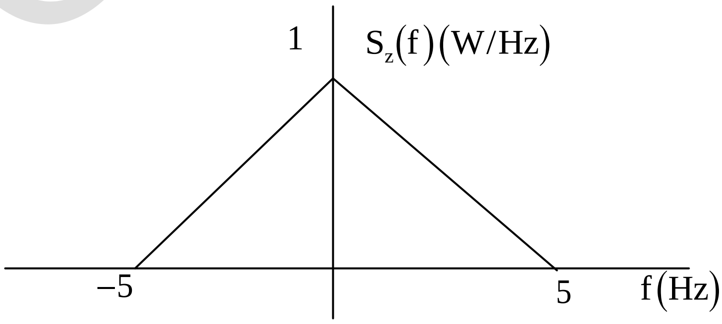
Solving (3) and (4), we have

$$C_1 + C_2 = 1$$
$$C_1 + 4C_2 = 7$$
$$-3C_2 = -6 \Rightarrow C_2 = 2 \ \therefore C_1 = -1$$

From (2),

$$y = (-1)x + (2)x^3$$
$$\Rightarrow y(1.5) = (-1)(1.5) + 2(1.5)^3$$
$$\Rightarrow y(1.5) = 5.25$$

50. Let a random process Y(t) be described as $Y(t) = h(t) \times X(t) + Z(t)$, where X(t) is a white noise process with power spectral density $S_X(f) = 5W/Hz$. The filter h(t) has a magnitude response given by $|H(f)| = 0.5$ for $-5 \le f \le 5$, and zero elsewhere. Z(t) is a stationary random process, uncorrelated with X(t), with power spectral density as shown in the figure.

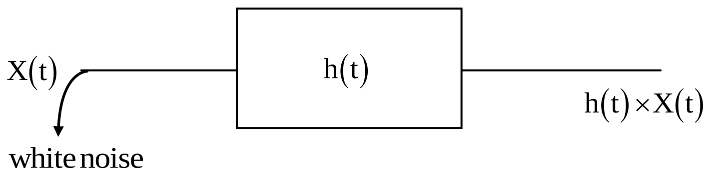


The power in Y(t), in watts, is equal to ________________W. (rounded off to two decimal places).

Key: (17.5)

Sol: Power in $y(t) = \begin{bmatrix}\text{Power in} \\ h(t)\times X(t)\end{bmatrix} + \left[\text{Power in } Z(t)\right]$

$\left(Z(t)\ \&\ X(t)\text{ uncorrelated}\right)$



Power in $h(t)\times X(t) = \int_{-\infty}^{\infty} |H(f)|^2 S_{XX}(f)df$

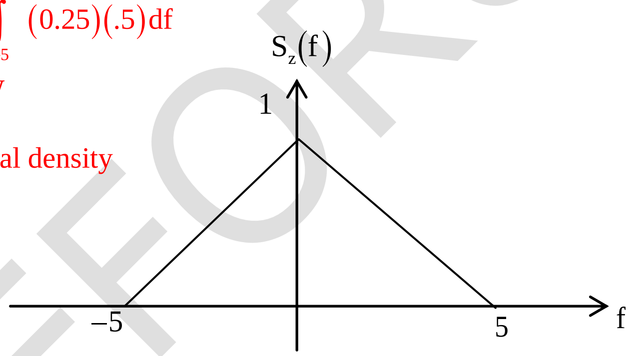
$= \int_{-\infty}^{\infty} |H(f)|^2 (5)df = \int_{-5}^{5} (0.25)(.5)df$

$= (10)(1.25) = 12.5W$

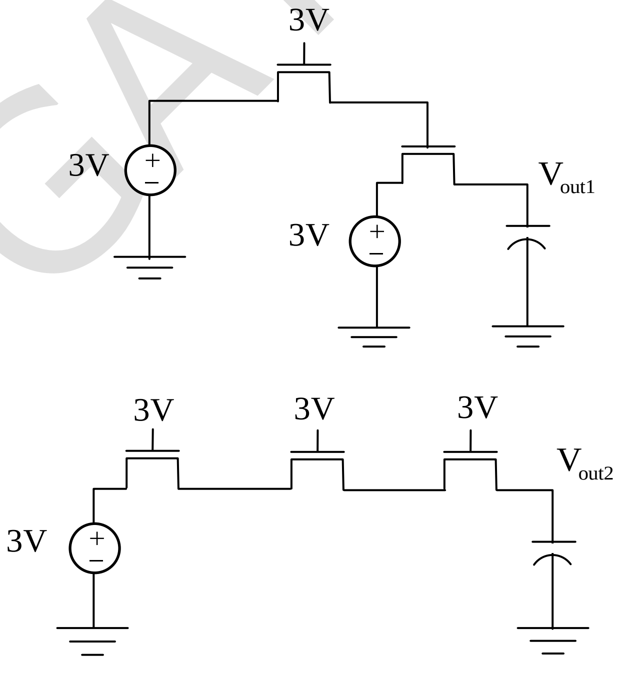
Power in $S_2(f) =$ Area under power spectral density

$P_{z(t)} = \left(\frac{1}{2}\right)(10)(1) = 5W$

$\therefore$ Power in y(t) = 12.5 + 5 = 17.5W



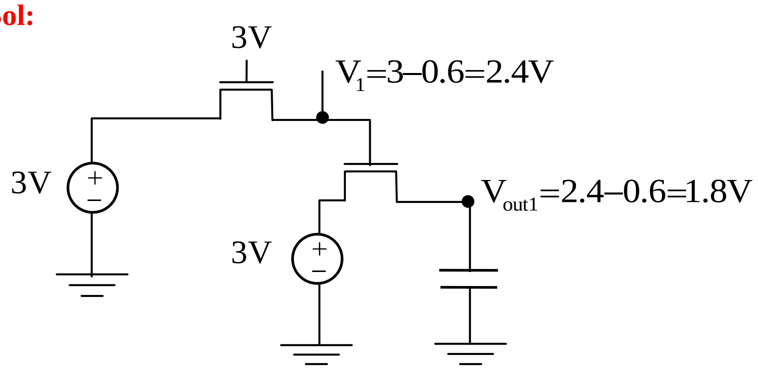
51. In the circuits shown, the threshold voltage of each nMOS transistor is 0.6V. Ignoring the effect of channel length modulation and body bias, the values of V_{out1} and V_{out2}, respectively, in volts, are

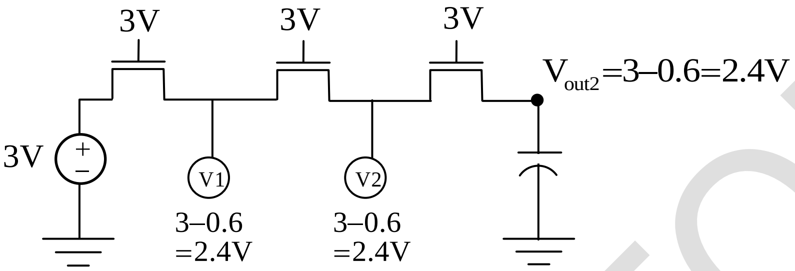


(A) 2.4 and 1.2 (B) 2.4 and 2.4 (C) 1.8 and 1.2 (D) 1.8 and 2.4

Key: (D)

Sol:



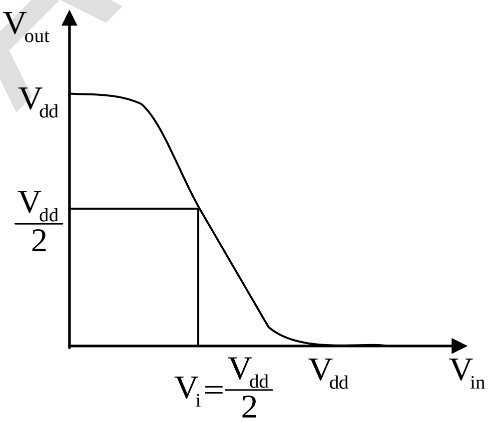


52. A CMOS inverter, designed to have a mid-point voltage V_I equal to half of V_{dd}, as shown in the figure, has the following parameters:

$V_{dd} = 3V$

$\mu_n C_{ox} = 100\mu A/V^2$; $V_{tn} = 0.7V$ for nMOS

$\mu_p C_{ox} = 40\mu A/V^2$; $|V_{tp}| = 0.9V$ for pMOS



The ratio of $\left(\frac{W}{L}\right)_n$ to $\left(\frac{W}{L}\right)_p$ is equal to ________. (rounded off to three decimal places).

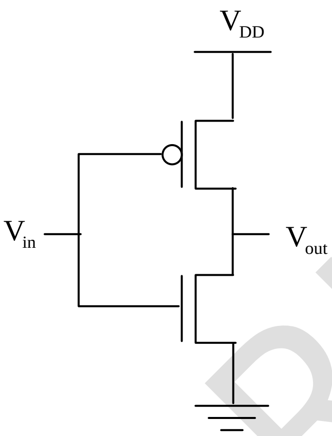
Key: (0.225)

Sol: At $V_i = \frac{V_{DD}}{2}$, both the MOSFETs are in saturation and both MOSFETs have the same current.

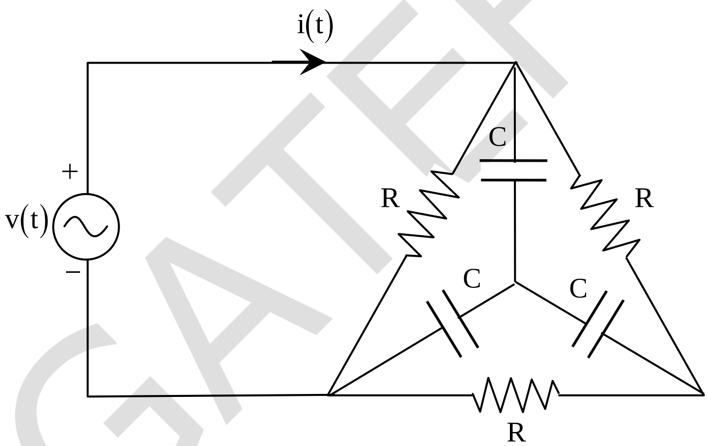
$$\therefore \left(\frac{1}{2}\right)\left(100\mu A / V^2\right)\left(\frac{W}{L}\right)_n \underbrace{[1.5-0.7]^2}_{}\ \ \left(\frac{V_{dd}}{2}\right)\uparrow$$

$$=\left(\frac{1}{2}\right)\left(40\mu A / V^2\right)\left(\frac{W}{L}\right)_p \left(3-1.5-0.9\right)^2$$

$$\frac{\left(\frac{W}{L}\right)_n}{\left(\frac{W}{L}\right)_p} = \frac{(40)(0.6)^2}{(100)(0.8)^2} = 0.225$$



53. In the circuit shown, if $v(t) = 2\sin(1000t)$ volts, $R = 1k\Omega$, and $C = 1\mu F$, then the steady-state current i(t), in milliamperes (mA) is



(A) $\sin(1000t) + \cos(1000t)$ (B) $\sin(1000t) + 3\cos(1000t)$

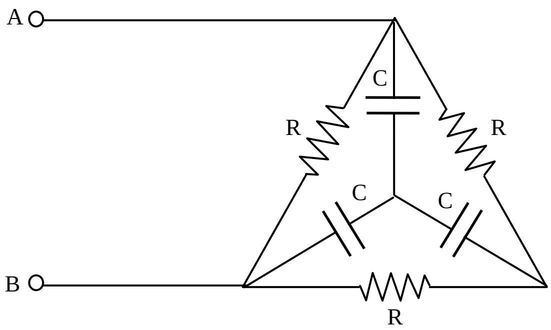
(C) $2\sin(1000t) + 2\cos(1000t)$ (D) $3\sin(1000t) + \cos(1000t)$

Key: (D)

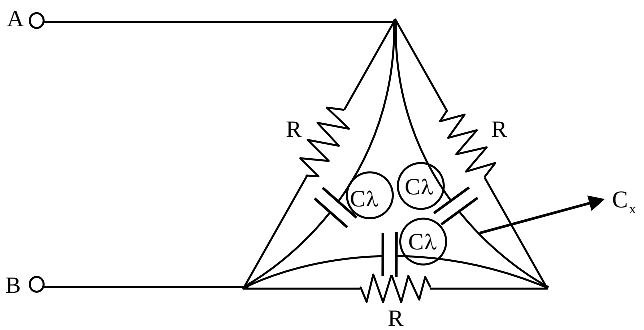
Sol: → It is given that $V(t) = 2\sin 100t = \bar{V} = 2\underline{|0°}$
$R = 1k\Omega,\ C = 1\mu F$

→ By observity the circuit we can say

$\bar{I} = \frac{\bar{V}}{Z}$

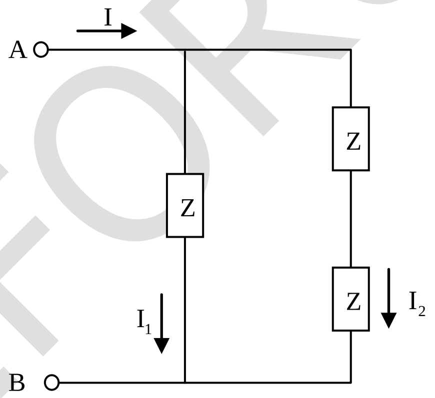


→ When each element of star network are same then its corresponding delta element are same and it becomes



$Z_\Delta = 3Z^*$, but in capacitor case $Z=\frac{1}{j\omega C}$, So if the capacitor of star network are C each then in its delta equivalent it becomes $C/3 = C_x$

→ The network can be further redrawn as



Where $Z = R \| \frac{1}{j\omega C_x} = \frac{R/jwC_x}{R+\frac{1}{j\omega C_x}} = \frac{R}{1+j\omega RC_x}$

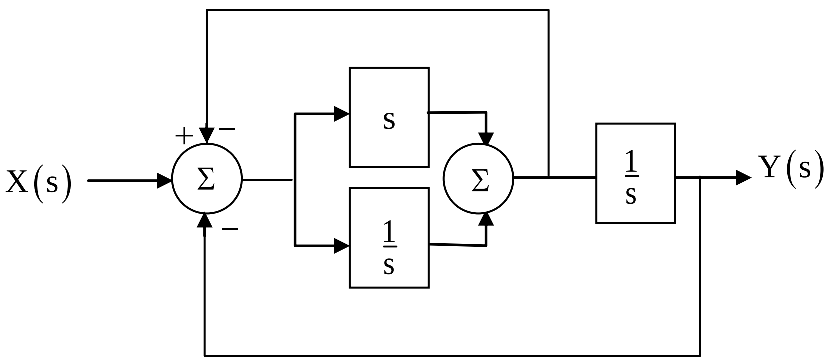
$$= \frac{R}{1+j\omega R\frac{C}{3}} = \frac{3R}{3+j\omega RC} = \frac{3R}{3+j}$$

$$\to \bar{I} = \bar{I}_1+\bar{I}_2 = \frac{\bar{V}}{Z}+\frac{\bar{V}}{2Z} = \frac{2\angle 0}{(3R/3+j)} + \frac{2\angle 0}{(65R/3+j)}$$

$$= \frac{3+j}{3R}[2\angle 0 + 1\angle 0] = \frac{3+j}{R}(1\angle 0) = \frac{[3\angle 0]+[1\angle 90]}{R}$$

$$= 3\angle 0 + 1\angle 90\,mA\ (\because 12 - 1000\Omega) = 3\sin 1000t + \sin(1000t+90°)$$

54. The block diagram of a system is illustrated in the figure shown, where X(s) is the input and Y(s) is the output. The transfer function $H(s)=\frac{Y(s)}{X(s)}$ is

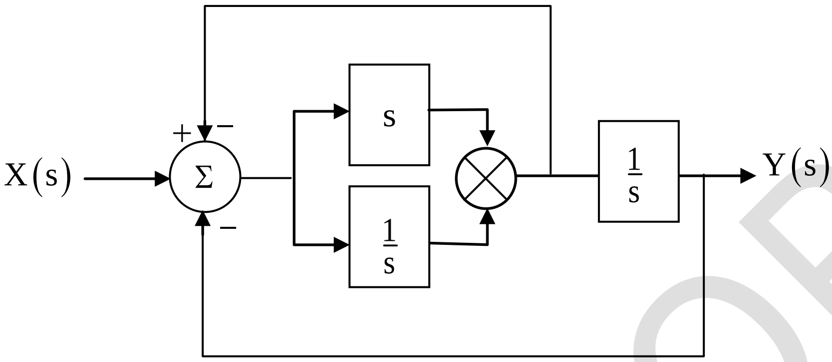


(A) $H(s)=\dfrac{s^2+1}{2s^2+1}$ (B) $H(s)=\dfrac{s^2+1}{s^3+2s^2+s+1}$

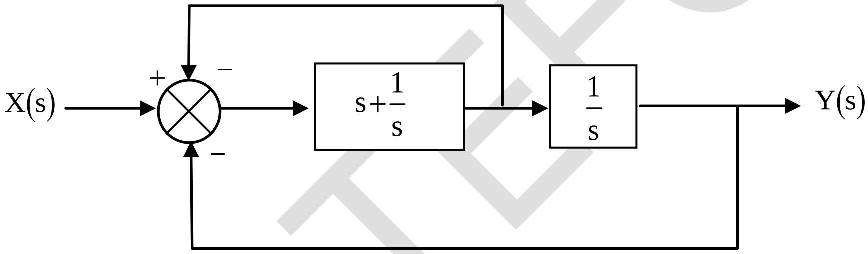
(C) $H(s)=\dfrac{s+1}{s^2+s+1}$ (D) $H(s)=\dfrac{s^2+1}{s^3+s^2+s+1}$

Key: (B)

Sol: Given block diagram



It can be reduced to

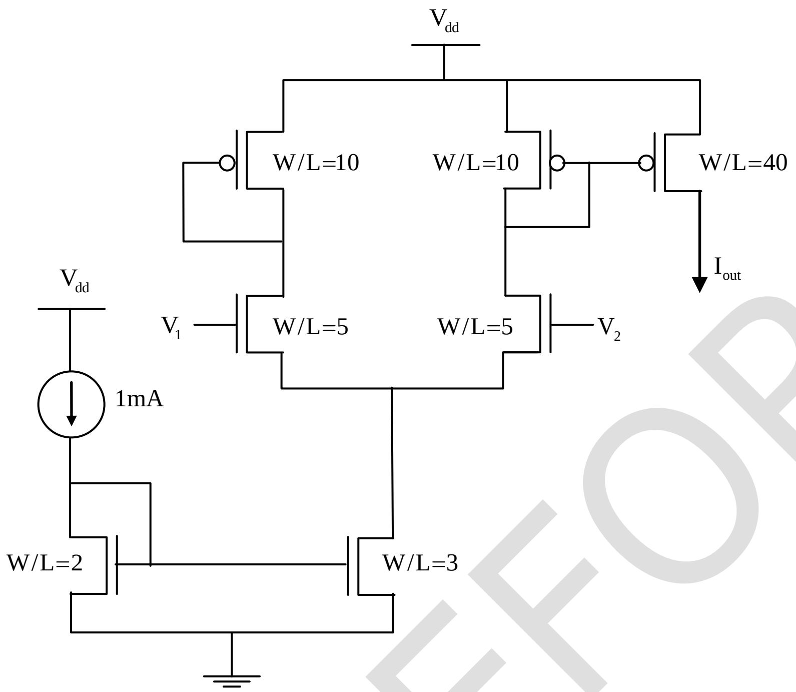


$$\frac{Y(s)}{X(s)}=\frac{\frac{(s^2+1)}{s}\times\frac{1}{s}}{1+\frac{s^2+1}{s}+\frac{s^2+1}{s^2}}=\frac{\frac{(s^2+1)}{s^2}}{\frac{s^2+(s^2+1)s+s^2+1}{s^2}}$$

$$=\frac{(s^2+1)}{s^2+s^3+s+s^2+1}=\frac{(s^2+1)}{s^3+2s^2+s+1}$$

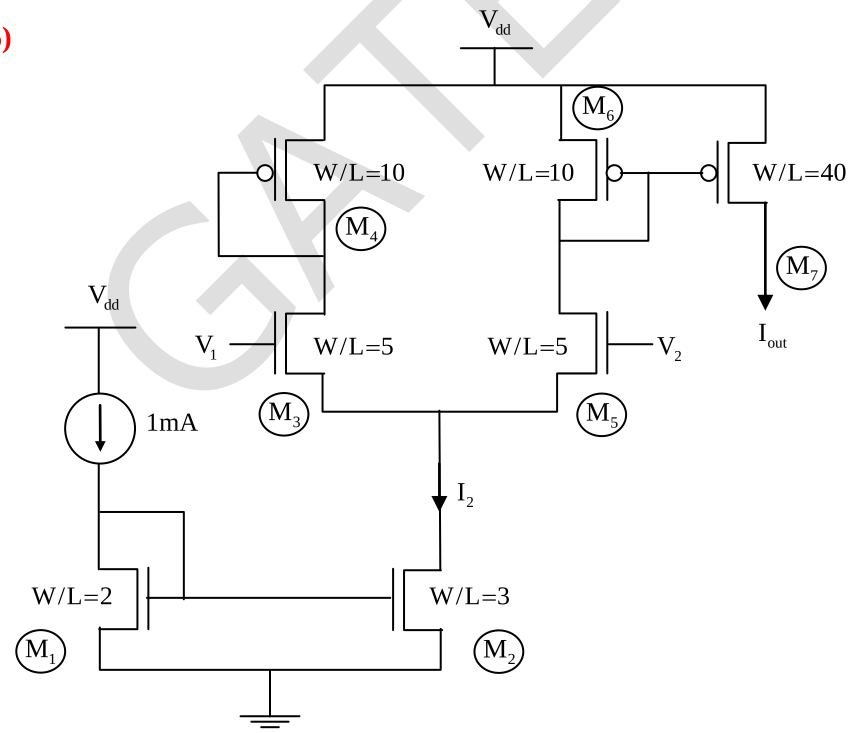
Hence, the transfer function $H(s)=\dfrac{Y(s)}{X(s)}$ is $\dfrac{(s^2+1)}{(s^3+2s^2+s+1)}$

55. In the circuit shown, $V_1 = 0$ and $V_2 = V_{dd}$. The other relevant parameters are mentioned in the figure. Ignoring the effect of channel length modulation and the body effect, the value of I_{out} is ____mA (rounded off to one decimal place).



Key: (6)

Sol:



M_1 and M_2 have the same V_{gs}

$\therefore$ Current flows in the ratio of W/L

$$\therefore I_2 = \left(\frac{3}{2}\right)^{(1mA)} = 1.5mA$$

$V_1 = 0;$ Therefore M_3 is in cut off and entire I_2 current flows through M_5 branch.

$$\therefore I_5 = 1.5mA$$

$$I_{out} = \left(\frac{40}{10}\right)^{[I_5]} \quad [\text{Ratio of } W/L]$$

$$\therefore I_{out} = 4 \times 1.5mA = 6mA$$