

GENERAL APTITUDE

1. If $\left(x - \frac{1}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2 = x + 2$, then the value of x is:

- (A) 6 (B) 2 (C) 8 (D) 4

Key: (D)

2. Pen: Write:: Knife: _____

Which one of the following options maintains a similar logical relation in the above?

- (A) Cut (B) Blunt (C) Sharp (D) Vegetables

Key: (A)

3. Six students P, Q, R, S, T and U, with distinct heights, compare their heights and make the following observations.

Observations I: S is taller than R.

Observation II: Q is the shortest of all

Observations III: U is taller than only one student.

Observations IV: T is taller than S but is not the tallest.

The number of students that are taller than R is the same as the number of students shorter than _____.

- (A) R (B) S (C) P (D) T

Key: (B)

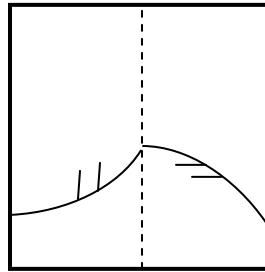
4. The number of students in three classes is in the ratio 3:13:6. If 18 students are added to each class, the ratio changes to 15:35:21

The total number of students in all the three classes in the beginning was:

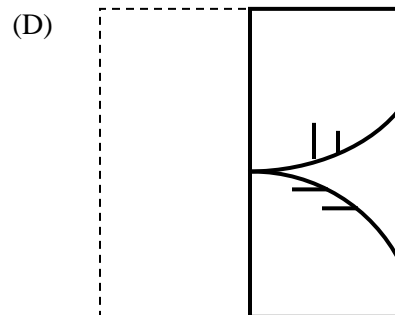
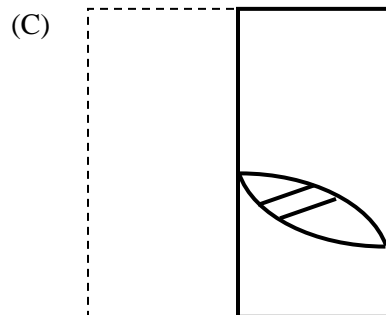
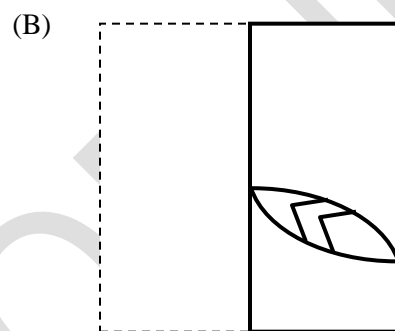
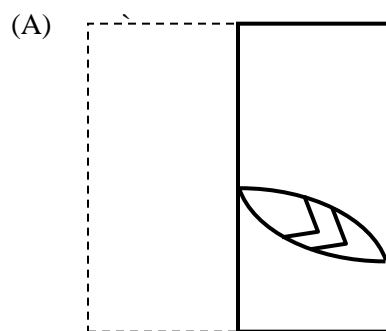
- (A) 66 (B) 88 (C) 110 (D) 22

Key: (B)

5.



A transparent square sheet shown above is folded along the dotted line. The folded sheet will look like _____.



Key: (A)

6. Gauri said that she can play the keyboard _____ her sister.

(A) as worse as

(B) as nicest as

(C) as well as

(D) as better as

Key: (C)

7. Listening to music during exercise improves exercise performance and reduces discomfort. Scientists researched whether listening to music while studying can help students learn better and the results were inconclusive. Students who needed external stimulation for studying fared worse while students who did not need any external stimulation benefited from music.

Which of the following statements is the CORRECT inference of the above passage?

- (A) Listening to music has a clear positive effect on physical exercise. Music has a positive effect on learning only in some students.
- (B) Listening to music has no effect on learning and a positive effect on physical exercise.
- (C) Listening to music has a clear positive effect on learning in all students. Music has a positive effect only in some students who exercise.
- (D) Listening to music has a clear positive effect both on physical exercise and on learning

Key: (A)

8.

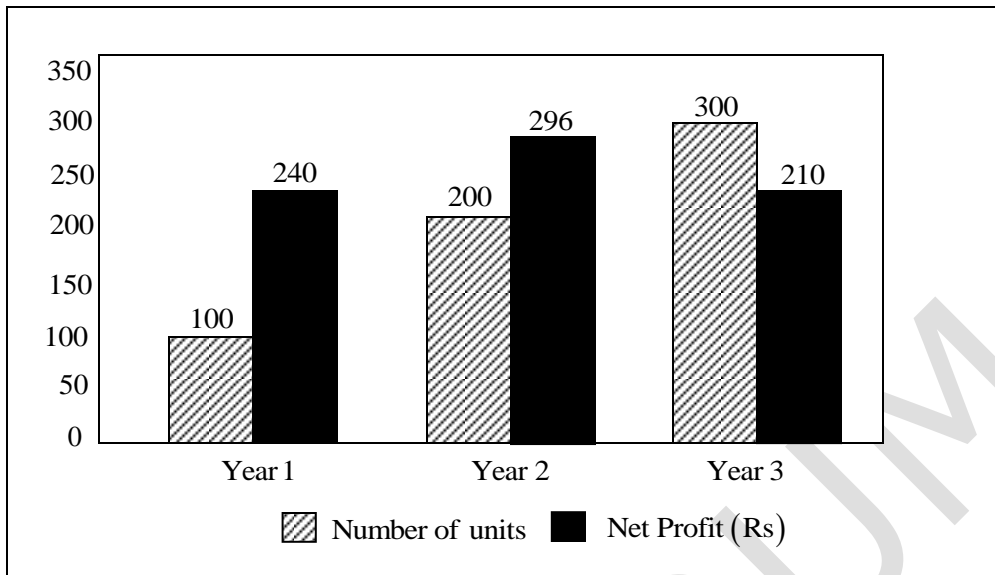


A jigsaw puzzle has 2 pieces. One of the pieces is shown above. Which one of the given options for the missing piece when assembled will form a rectangle? The piece can be moved, rotated, or flipped to assemble with the above piece.



Key: (C)

9.



The number of units of a product sold in three different years and the respective net profits are presented in the figure above. The cost/unit in Year 3 was Rs 1, which was half the cost/unit in Year 2. The cost/unit in Year 3 was one-third of the cost/unit in Year 1. Taxes were paid on the selling price at 10%, 13% and 15% respectively for the three years. Net profit is calculated as the difference between the selling price and the sum of cost and taxes paid in that year.

The ratio of the selling price in Year 2 to the selling price in Year 3 is _____.

- (A) 1:1 (B) 3:4 (C) 1:2 (D) 4:3

Key: (D)

10. If θ is the angle, in degrees, between the longest diagonal of the cube and any one of the edges of the cube, then, $\cos \theta =$

- (A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

Key: (D)

COMPUTER SCIENCE ENGINEERING

1. Consider the following multi-threaded code segment (in a mix of C and pseudo-code), invoked by two processes P1 and P2, and each of the processes spawns two threads T1 and T2:

```
int x = 0;           //global
Lock L1;           //global
main( ) {
    create a thread to execute foo( );           //Thread T1
    create a thread to execute foo( );           //Thread T2
    wait for the two threads to finish execution;
    print (x);}

foo( ) {
int y = 0
Acquire L1;
x = x + 1;
y = y + 1;
Release L1;
print (y); }
```

Which of the following statements(s) is/are correct?

- (A) Both T1 and T2, in both the processes, will print the value of y as 1
- (B) Both P1 and P2 will print the value of x as 2
- (C) At least one of the threads will print the value of y as 2
- (D) At least one of P1 and P2 will print the value of x as 4

Key: 0

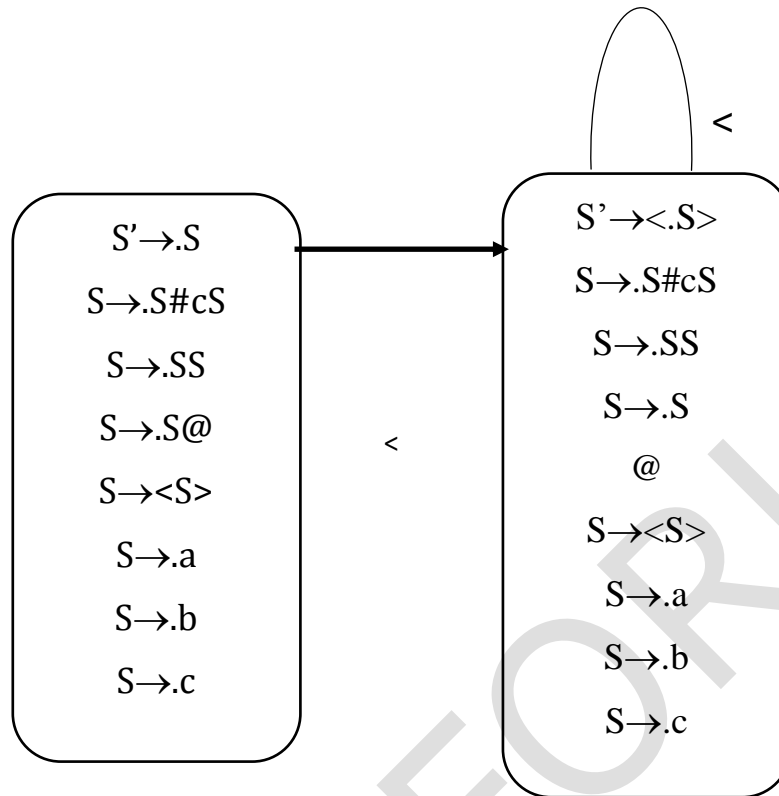
2. Consider the following augmented grammar with {#, @, <, >, a, b, c} as the set of terminals.

```
S' → S
S → S#cS
S → SS
S → S@
S → <S>
S → a
S → b
S → c
```

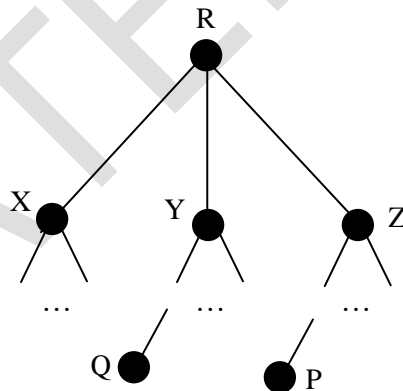
Let $I_0 = \text{CLOSURE}(\{S' \rightarrow \bullet S\})$. The number of items in the set $\text{GOTO}(\text{GOTO}(I_0, <), <)$ is _____.

Key: (8)

Sol:



3. Consider a computer network using the distance vector routing algorithm in its network layer. The partial topology of the network is as shown below.



The objective is to find the shortest-cost path from the router R to routers P and Q. Assume that R does not initially know the shortest routes to P and Q. Assume that R has three neighbouring routers denoted as X, Y and Z. During one iteration, R measures its distance to its neighbours X, Y and Z as 3, 2 and 5, respectively. Router R gets routing vectors from its neighbours that indicate that the distance to router P from routers X, Y and Z are 7, 6 and 5, respectively. The routing vector also indicates that the distance to router Q from routers X, Y and Z are 4, 6 and 8, respectively. Which of the following statement(s) is/are correct with respect to the new routing table of R, after updation during this iteration?

- (A) The next hop router for a packet from R to P is Y
- (B) The distance from R to Q will be stored as 7
- (C) The distance from R to P will be stored as 10
- (D) The next hop router for a packet from R to Q is Z

Key: (A,B)

Sol: The distance from R to Q is 7 via X.

The distance from R to P is 8 via Y.

4. A bag has r red balls and b black balls. All balls are identical except for their colours. In a trial, a ball is randomly drawn from the bag, its colour is noted and the ball is placed back into the bag along with another ball of the same colour. Note that the number of balls in the bag will increase by one, after the trial. A sequence of four such trials is conducted. Which one of the following choices gives the probability of drawing a red ball in the fourth trial?

- (A) $\left(\frac{r}{r+b}\right)\left(\frac{r+1}{r+b+1}\right)\left(\frac{r+2}{r+b+2}\right)\left(\frac{r+3}{r+b+3}\right)$
- (B) $\frac{r}{r+b}$
- (C) $\frac{r}{r+b+3}$
- (D) $\frac{r+3}{r+b+3}$

Key: (*) or (B)

Sol:

Total balls before 1st trial is $r + b$
 Total balls before 2nd trial is $r + b + 1$
 Total balls before 3rd trial is $r + b + 2$
 Total balls before 4th trial is $r + b + 3$ } i.e., in the first three trials, a black ball is drawn

∴ P(a red ball is in the 4th trial)
 = P(black ball in 1st trial) × P (black ball in 2nd trial) × P (black ball in 3rd trial) × P(red ball is 4th trial)

$$= \left(\frac{b}{r+b}\right)\left(\frac{b+1}{r+b+1}\right)\left(\frac{b+2}{r+b+2}\right)\left(\frac{r}{r+b+3}\right)$$

No option is matching.

(OR)—B

Assume that after i -th iteration, we have r red balls and b black balls.

So, probability of choosing red ball in $(i+1)$ th iteration will be = $[r/(r+b)]$

So, probability of choosing red ball in $(i+2)$ th iteration will be :

(note that, in $(i+1)$ th iteration, we could either pick black or red ball.)

$$[r/(r+b)][(r+1)/(r+b+1)] + [b/(r+b)][r/(r+b+1)] = [r/(r+b)]$$

So, in the beginning we have r red balls, and b green balls. So, in every iteration, the probability of choosing a red ball will be $[r/(r+b)]$.

5. In an examination a student can choose the order in which two questions (Ques A and Ques B) must be attempted.
- If the first questions is answered wrong, the student gets zero marks.
 - If the first question is answered correctly and the second question is not answered correctly, the student gets the marks only for the first question.
 - If both the questions are answered correctly, the student gets the sum of the marks of the two questions.

The following table shows the probability of correctly answering a question and the marks of the question respectively.

question	probability of answering correctly	marks
QuesA	0.8	10
QuesB	0.5	20

Assuming that the student always wants to maximize her expected marks in the examination, in which order should she attempt the questions and what is the expected marks for that order (assume that the questions are independent)?

- (A) First QuesB and then QuesA. Expected marks 22
- (B) First QuesA and then QuesB. Expected marks 16
- (C) First QuesA and then QuesB. Expected marks 14
- (D) First QuesB and the QuesA. Expected marks 14

Key: (B)

6. Consider the following statements S1 and S2 about the relational data model:

S1: A relation scheme can have at most one foreign key.

S2: A foreign key in a relation scheme R cannot be used to refer to tuples of R.

Which one of the following choices is correct?

- (A) Both S1 and S2 are true
- (B) S1 is false and S2 is true
- (C) Both S1 and S2 are false
- (D) S1 is true and S2 is false

Key: (C)

Sol: Foreign key : Set of fields in one relation that is used to 'reference' a tuple in another (or same) relation. A relation schema can have more than one foreign key. Also, a foreign key in a relation can 'reference' a tuple in same relation.

7. What is the worst-case number of arithmetic operations performed by recursive binary search on a sorted array of size n ?

- (A) $\Theta(\sqrt{n})$ (B) $\Theta(n)$ (C) $\Theta(\log_2(n))$ (D) $\Theta(n^2)$

Key: (C)

Sol: In worst case it will take $\theta(\log n)$ time (when element is not present in the array).

8. Let S be the following schedule of operations of three transactions T_1, T_2 and T_3 in a relational database system:

$$R_2(Y), R_1(X), R_3(Z), R_1(Y), W_1(X), R_2(Z), W_2(Y), R_3(X), W_3(Z)$$

Consider the statements P and Q below:

P: S is conflict-serializable.

Q: If T_3 commits before T_1 finishes, then S is recoverable

Which one of the following choices is correct?

- (A) Both P and Q are false (B) P is false Q is true
(C) P is true and Q is false (D) Both P and Q are true

Key: (C)

Sol: As there is no cycle in precedence graph so, S is conflict serializable.

If T_3 commits after T_1 then only recoverability is possible.

9. For a string ω , we define ω^R to be the reverse of ω . For example, if $\omega = 01101$ then $\omega^R = 10110$.

Which of the following languages is/are context-free?

- (A) $\{\omega x x^R \omega^R \mid \omega, x \in \{0,1\}^*\}$ (B) $\{\omega x \omega^R \mid \omega, x \in \{0,1\}^*\}$
(C) $\{\omega x \omega^R x^R \mid \omega, x \in \{0,1\}^*\}$ (D) $\{\omega \omega^R x x^R \mid \omega, x \in \{0,1\}^*\}$

Key: (A,B,D)

- Sol:**
- $\{w x w^R x^R \mid w, x \in \{0,1\}^*\}$ is context sensitive language
 - $\{w w^R x x^R \mid w, x \in \{0,1\}^*\}$ is **context free language**, Single stack can be used for pushing w and popping w^R . Then we can push x and pop x^R .
 - $\{w x w^R \mid w, x \in \{0,1\}^*\}$ is regular language hence **context free**. The regular expression is $(0+1)^*$
 - $\{w x x^R w^R \mid w, x \in \{0,1\}^*\}$ is **context free**, We can push w as well as x and pop x^R and w^R .

10. The relation scheme given below is used to store information about the employees of a company, where empId is the key deptId indicates the department to which the employee is assigned. Each employee is assigned to exactly one department.

```
emp(empId, name, gender, salary, deptId)
```

Consider the following SQL query:

```
select deptId, count (*)  
from emp  
where gender = "female" and salary > (select avg(salary) from emp)  
group by deptId;
```

The above query gives, for each department in the company, the number of female employees whose salary is greater than the average salary of

- (A) employees in the department (B) female employees in the department
(C) female employees in the company (D) employees in the company

Key: (D)

Sol: The above query gives, for each department in the company, the number of female employees whose salary is greater than the average salary of all employees in the company.

11. Consider a complete binary tree with 7 nodes. Let A denote the set of first 3 elements obtained by performing Breadth-First Search (BFS) starting from the root. Let B denote the set of first 3 elements obtained by performing Depth-First Search (DFS) starting from the root.

The value of $|A - B|$ is _____.

Key: (1)

If the given CBT has root node A, its children B,C, and children of B are D,E then BFS will visit {A,B,C} as first three nodes, and DFS will visit {A,B,D}, so,

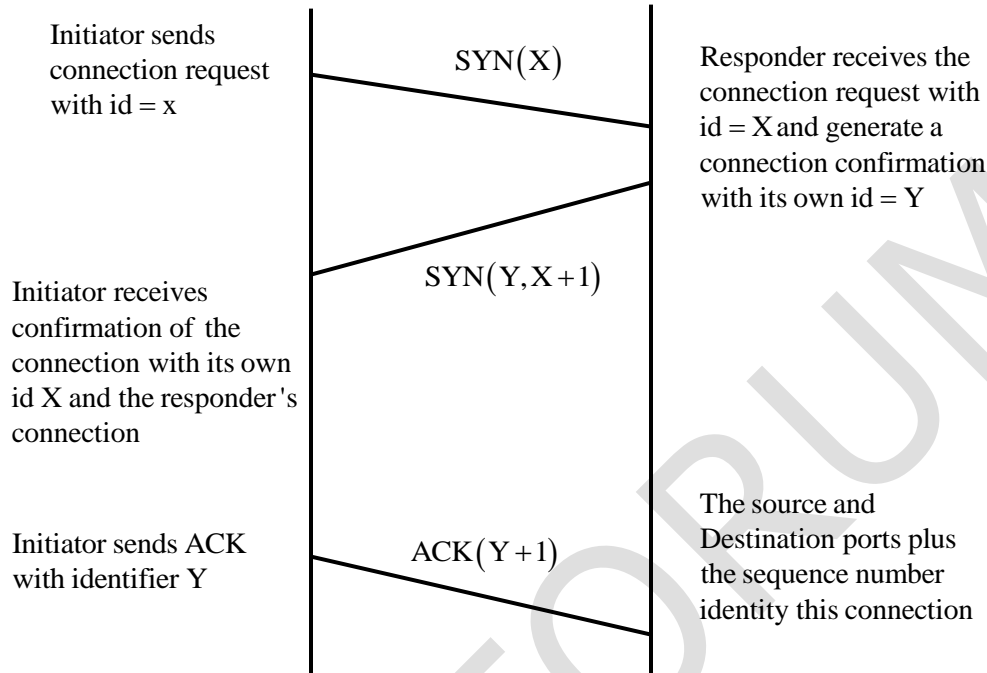
$$|\{A,B,C\} - \{A,B,D\}| = 1$$

12. Consider the three-way handshake mechanism followed during TCP connection establishment hosts P and Q. Let X and Y be two random 32-bit starting sequence numbers chosen by P and Q respectively. Suppose P sends a TCP connection request message to Q with a TCP segment having SYN bit = 1, SEQ number = X, and ACK bit = 0. Suppose Q accepts the connection request. Which one of the following choices represents the information present in the TCP segment header that is sent by Q to P?

- (A) SYN bit = 0, SEQ number = X + 1, ACK bit = 0, ACK number = Y, FIN bit = 1
(B) SYN bit = 1, SEQ number = Y, ACK bit = 1, ACK number = X + 1, FIN bit = 0
(C) SYN bit = 1, SEQ number = Y, ACK bit = 1, ACK number = X, FIN bit = 0
(D) SYN bit = 1, SEQ number = X + 1, ACK bit = 0, ACK number = Y, FIN bit = 0

Key: (B)**Sol:**

TCP (Transport Connection Establishment)



13. A data file consisting of 1,50,000 student-records is stored on a hard disk with block size of 4096 bytes. The data file is sorted on the primary key RollNo. The size of a record pointer for this disk is 7 bytes. Each student-record has a candidate key attribute called ANum of size 12 bytes. Suppose an index file with records consisting of two fields, ANum value and the record pointer to the corresponding student record, is built and stored on the same disk. Assume that the records of data file and index file are not split across disk blocks. The number of blocks in the index file is _____.

Key: (698)**Sol:** Block factor = $\lceil 4096/19 \rceil = 215$ Number of index blocks = $\lceil 1,50,000 / 215 \rceil = 698$ blocks

14. Consider the following ANSI C program:

```
#include <stdio.h>
#include <stdlib.h>

Struct Node{
    Int value;
    Struct Node *next;};

Int main ( ) {
    Struct Node *boxE, *head, *boxN; int index = 0;
    boxE = head = (struct Node*) malloc (sizeof (struct Node));
    head->value = index;
    For (index = 1; index <=3; index++) {
        box N = (struct Node *) malloc (sizeof (struct Node));
        BoxE->next = boxN;
        boxN->value = index;
        boxE= boxN; }
    for (index = 0; index <=3; index++) {
        printf("Value at index %d is %d\n", index, head->value);
        head = head->next;
        printf ("Value at index %d is %d\n", index + 1, head->value);}}
```

Which one of the statements below is correct about the program?

- (A) It dereferences an uninitialized pointer that may result in a run-time error
- (B) It has a missing return which will reported as an error by the compiler
- (C) Upon execution, the program creates a linked-list of five nodes
- (D) Upon execution, the program goes into an infinite loop

Key: (A)

Sol: It will result in run time error because it dereferences an uninitialized pointer in the loop when index value is 3.

17. Choose the correct choice(s) regarding the following propositional logic assertion S:

$$S: ((P \wedge Q) \rightarrow R) \rightarrow ((P \wedge Q) \rightarrow (Q \rightarrow R))$$

- (A) S is a tautology
- (B) The antecedent of S is logically equivalent to the consequent of S
- (C) S is a contradiction
- (D) S is neither a tautology nor a contradiction

Key: (A,B)

Sol: The antecedent of S is $(P \wedge Q) \rightarrow R$. Which can be written as $P' \vee Q' + R$

The consequent of S is $(P \wedge Q) \rightarrow (Q \rightarrow R)$ Which can be written as $P' \vee Q' + Q' + R$ which is $P' \vee Q' + R$

So, antecedent and consequent, both are equivalent.

18. Suppose we want to design a synchronous circuit that processes a string of 0's and 1's. Given a string, it produces another string by replacing the first 1 in any subsequence of consecutive 1's by a 0. Consider the following example.

Input sequence: 00100011000011100

Output sequence: 00000001000001100

A Mealy Machine is a state machine where both the next state and the output are functions of the present state and the current input.

The above mentioned circuit can be designed as a two-state Mealy machine. The states in the Mealy machine can be represented using Boolean values 0 and 1. We denote the current state, the next state, the next incoming bit, and the output bit of the Mealy machine by the variables s, t, b and y respectively.

Assume the initial state of the Mealy machine is 0.

What are the Boolean expressions corresponding to t and y in terms of a and b?

- | | | | |
|-----------------|----------------|-----------------|-------------|
| (A) $t = s + b$ | (B) $t = b$ | (C) $t = s + b$ | (D) $t = b$ |
| $y = sb$ | $y = \bar{s}b$ | $y = \bar{s}b$ | $y = sb$ |

Key: (A)

19. Let H be a binary min-heap consisting of n elements implemented as an array. What is the worst case time complexity of an optimal algorithm to find the maximum element in H?

- (A) $\Theta(\log n)$
- (B) $\Theta(n \log n)$
- (C) $\Theta(n)$
- (D) $\Theta(1)$

Key: (A)

The maximum element can be present anywhere in leaf level of the min heap, so, we need to have $n/2$ comparisons because we have $n/2$ leaf nodes in minheap of n elements. So, answer is $O(n)$.

20. If x and y are two decimal digits and $(0.1101)_2 = (0.8xy5)_{10}$, the decimal value of $x + y$ is _____.

Key: (3)

Sol: It is given that $(0.1101)_2 = (0.8xy5)_{10}$, we need to obtain decimal value of $x + y$.

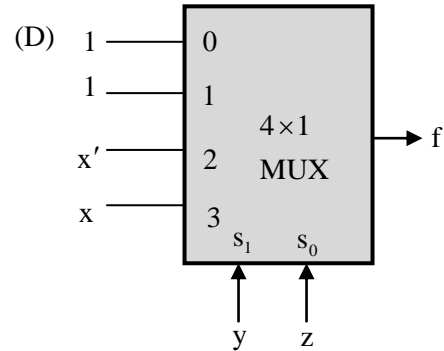
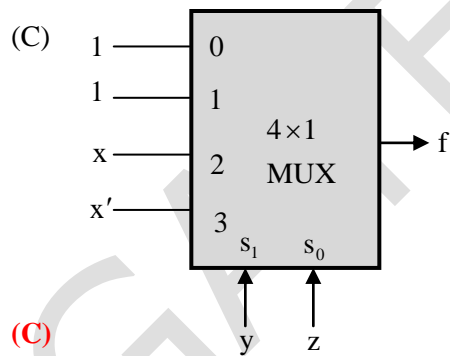
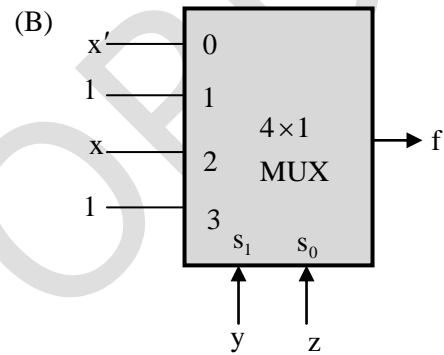
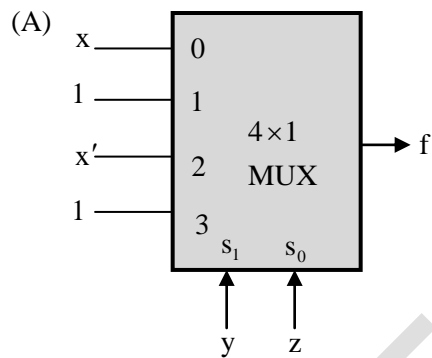
$$(0.1101)_2 = (1 \times 2^{-1}) + (1 \times 2^{-2}) + (0 \times 2^{-3}) + (1 \times 2^{-4}) = \frac{1}{2} + \frac{1}{4} + 0 + \frac{1}{16} = \frac{8+4+1}{16} = \frac{13}{16} = 0.8125$$

$$\Rightarrow (0.1101)_2 = (0.8125)_{10} = (0.8xyz)_{10}$$

By comparison $x = 1, y = 2$ so $x + y = 3$.

21. Which one of the following circuits implements the Boolean function given below?

$f(x, y, z) = m_0 + m_1 + m_3 + m_4 + m_5 + m_6$, where m_i is the i^{th} minterm.



Key: (C)

Sol: A multiplexer based circuit to be implement of where $f(x, y, z) = \sum m(0, 1, 3, 4, 5, 6)$, we need to select the correct design.

Option (A)

$$f = \bar{y} \bar{x} + \bar{y} z + y \bar{z} \bar{x} + y z = \sum m(1, 2, 3, 4, 5, 7)$$

$$= x \bar{y} \bar{z} + \bar{y} z + \bar{x} y \bar{z} + y z$$

$$= x \bar{y} \bar{z} + \bar{y} z + \bar{x} y \bar{z} + \bar{y} z$$

$$\begin{matrix} \textcircled{100} & \textcircled{001} & \textcircled{010} & \textcircled{011} \\ m_4 & & m_2 & m_3, m_7 \end{matrix}$$

So option A is not correct.

Option (B)

$$f = \bar{x} \bar{y} \bar{z} + \bar{y}z + y\bar{z} + yz = \Sigma m(0,1,3,5,6,7)$$

$$= \bar{x} \bar{y} \bar{z} + \bar{y}z + xy\bar{z} + yz$$

000	001	110	011
m_0	m_1, m_5	m_6	m_3, m_7

So option B is not correct.

Option (C)

$$f = \bar{y} \bar{z} + \bar{y}z + xy\bar{z} + \bar{x}yz = \Sigma m(0,1,3,4,5,6)$$

$$= \bar{y} \bar{z} + \bar{y}z + xy\bar{z} + \bar{x}yz$$

000	001	110	011
m_0, m_4	m_1, m_5	m_6	m_3

So option C is not correct.

Option (D)

$$f = \bar{y} \bar{z} + \bar{y}z + \bar{x}y\bar{z} + xyz = \Sigma m(0,1,2,4,5,7)$$

$$= \bar{y} \bar{z} + \bar{y}z + \bar{x}y\bar{z} + xyz$$

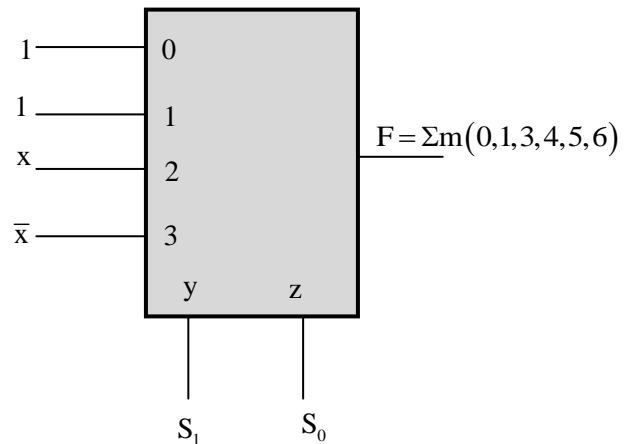
000	001	010	111
m_0, m_4	m_1, m_5	m_2	m_7

So option D is not correct.

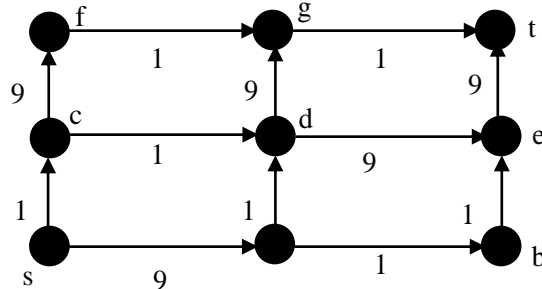
Method 2:

If y, z are in select lines x should be data line

	I_0	I_1	I_2	I_3
\bar{x}	0	1	2	3
x	4	5	6	7
	1	1	x	\bar{x}



22. In a directed acyclic graph with a source vertex s , the quality-score of a directed path is defined to be the product of the weights of the edges on the path. Further, for a vertex v other than s , the quality-score of v is defined to be maximum among the quality-scores of all the paths from s to v . The quality-score of s is assumed to be 1.



The sum of the quality-scores of all the vertices in the graph shown above is _____.

Key: (929)

Sol: Quality scores of the vertices are as follows :

$$QS(S) = 1 \text{ (Given)}$$

$$QS(C) = 1 ; QS(A) = 9$$

$$QS(D) = 9 ; QS(B) = 9 ; QS(E) = 9 \times 9 = 81$$

$$QS(F) = 9 ; QS(G) = 81 ; QS(T) = 81 \times 9 = 729$$

Total is 929.

23. For a statement S in a program, in the context of liveness analysis, the following sets are defined:

USE (S): the set of variables used in S

IN (S): the set of variables that are live at the entry of S

OUT (S): the set of variables that are live at the exit of S

Consider a basic block that consists of two statements, S_1 followed by S_2 .

Which one of the following statements is correct?

(A) $OUT(S_1) = IN(S_1) \cup USE(S_1)$

(B) $OUT(S_1) = IN(S_2)$

(C) $OUT(S_1) = USE(S_1) \cup IN(S_2)$

(D) $OUT(S_1) = IN(S_2) \cup OUT(S_2)$

Key: (B)

Sol: The variable live at the end of the block S_1 will be IN for the next Block S_2

24. Consider a pipelined process with 5 stages, Instruction Fetch (IF), Instruction Decode (ID), Execute (EX), Memory Access (MEM), and Write Back (WB). Each stage of the pipeline, except the EX stage, takes one cycle. Assume that the ID stage merely decodes the instruction and the register read is performed in the Ex stage. The EX stage takes one cycle for ADD instruction and two cycles for MUL instruction. Ignore pipeline register latencies.

Consider the following sequence of 8 instructions.

ADD, MUL, ADD, MUL, ADD, MUL, ADD, MUL

Assume that every MUL instruction is data-dependent on the ADD instruction just before it and every ADD instruction (except the first ADD) is data-dependent on the MUL instruction just before it. The Speedup is defined as follows.

$$\text{Speed up} = \frac{\text{Execution time without operand forwarding}}{\text{Execution time with operand forwarding}}$$

The speedup achieved in executing the given instruction sequence on the pipeline processor (round to 2 decimal places) is _____.

Key: (1.875)

Sol: Execution time without operand forwarding

	IF	ID	EX	M	WB								
ADD													
MUL		IF	ID			EX	EX	M	WB				
ADD			IF	ID						EX	M	WB	
MUL													
ADD													
MUL													
ADD													
MUL													
ADD													

The First add instruction completes at 5 cycles, second dependent instruction will start execution phase after Write back (Register Read happens at Execute stage)

Instruction	Completion cycle No
ADD	5
MUL	8
ADD	12
MUL	16
ADD	19
MUL	23
ADD	26
SUB	30

26. Consider the following ANSI C program:

```
int main( ) {  
    integer x;  
    return 0;  
}
```

Which one of the following phases in a seven-phase C compiler will throw an error?

- (A) Syntax analyzer (B) Lexical analyzer
(C) Semantic analyzer (D) Machine dependent optimizer

Key: (A)

Sol: Integer is not a keyword in C language but a valid identifier. This error will be caught in syntax analysis phase

27. Consider a computer system with DMA support. The DMA module is transferring one 8-bit character in one CPU cycle from a device to memory through cycle stealing at regular intervals. Consider a 2 MHz processor. If 0.5% processor cycles are used for DMA, the data transfer rate of the device is _____ bit per second.

Key: (80000)

Sol: In One sec number of cycle generated by the CPU is 2×10^6

Since 0.5% cycles are used By DMA

The number of cycle used by the DMA is

$$= 2 \times 10^6 \times \frac{0.5}{100}$$

$$= 2 \times 10^6 \times \frac{1}{200}$$

$$= 10^4$$

In 1 cycle 8 bits transferred

$$\therefore 10^4 \text{ cycles} \text{ ----- } 10^4 \times 8 = 80000 \text{ bits}$$

28. Assume a two-level inclusive cache hierarchy, L1 and L2, where L2 is the larger of the two. Consider the following statements.

S₁ : Read misses in a write through L1 cache do not result in write backs of dirty lines to the L2.

S₂ : Write allocate policy must be used in conjunction with write through caches and no-write allocate policy is used with writeback caches.

Which of the following statements is correct?

(A) S_1 is false and S_2 is true

(B) S_1 is true and S_2 is true

(C) S_1 is false and S_2 is false

(D) S_1 is true and S_2 is false

Key: (D)

Sol: **Statement 1:**

Write Through - the information is written to both the block in the cache and to the block in the lower-level memory.

Advantage:

- read miss never results in writes to main memory
- easy to implement
- main memory always has the most current copy of the data (consistent)

Disadvantage:

- write is slower
- every write needs a main memory access
- as a result uses more memory bandwidth

Write back - the information is written only to the block in the cache.

The modified cache block is written to main memory only when it is replaced. To reduce the frequency of writing back blocks on replacement, a dirty bit is commonly used. This status bit indicates

whether the block is dirty (modified while in the cache) or clean (not modified). If it is clean the block is not written on a miss.

Advantage:

- writes occur at the speed of the cache memory
- multiple writes within a block require only one write to main memory
- as a result uses less memory bandwidth

Disadvantage:

- harder to implement
- main memory is not always consistent with cache
- reads that result in replacement may cause writes of dirty blocks to

main memory

Statement 2:

There are two common options on a write miss:

Write Allocate - the block is loaded on a write miss, followed by the write-hit action.

No Write Allocate - the block is modified in the main memory and not loaded into the cache.

Although either write-miss policy could be used with write through or write back, write-back caches generally use write allocate (hoping that subsequent writes to that block will be captured by the cache) and write-through caches often use no-write allocate (since subsequent writes to that block will still have to go to memory).

OR

S1 is False, in write through policy both cache and main memory updated simultaneously hence there is no Dirty lines will be present in the cache.

S2 is false, A write-back cache is more complex to implement, since it needs to track which of its locations have been written over, and mark them as *dirty* for later writing to the backing store. The data in these locations are written back to the backing store only when they are evicted from the cache, an effect referred to as a *lazy write*. For this reason, a read miss in a write-back cache (which requires a block to be replaced by another) will often require two memory accesses to service: one to write the replaced data from the cache back to the store, and then one to retrieve the needed data.

Other policies may also trigger data write-back. The client may make many changes to data in the cache, and then explicitly notify the cache to write back the data.

Since no data is returned to the requester on write operations, a decision needs to be made on write misses, whether or not data would be loaded into the cache. This is defined by these two approaches:

Write allocate (also called *fetch on write*): data at the missed-write location is loaded to cache, followed by a write-hit operation. In this approach, write misses are similar to read misses.

No-write allocate (also called *write-no-allocate* or *write around*): data at the missed-write location is not loaded to cache, and is written directly to the backing store. In this approach, data is loaded into the cache on read misses only.

Both write-through and write-back policies can use either of these write-miss policies, but usually they are paired in this way:

A write-back cache uses write allocate, hoping for subsequent writes (or even reads) to the same location, which is now cached.

A write-through cache uses no-write allocate. Here, subsequent writes have no advantage, since they still need to be written directly to the backing store

29. Which of the following regular expression represent(s) the set of all binary numbers that are divisible by three? Assume that the string ϵ is divisible by three.

(A) $(0^*(1(01^*0)^*1)^*)^*$

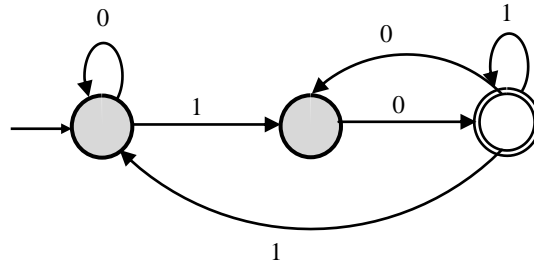
(B) $(0+11+11(1+00)^*00)^*$

(C) $(0+1(01^*0)^*1)^*$

(D) $(0+11+10(1+00)^*01)^*$

Key: (A, C)

Sol: The DFA accepting all binary number divisible by 3 is given by



The regular expression for the above DFA (direct method, considering two loops)

$(0+1(01^*0)^*1)^*$, which goes with option 3

We know that $(a+b)^*$ can also equal to $(a^*b^*)^*$

Hence $(0+1(01^*0)^*1)^* = (0^*(1(01^*0)^*1)^*)^*$

Both 2 and 3 are correct

30. Consider the following two statements about regular languages:

S_1 : Every infinite regular language contains an undecidable language as a subset.

S_2 : Every finite language is regular.

Which one of the following choices is correct?

(A) Both S_1 and S_2 are true

(B) Neither S_1 nor S_2 is true

(C) Only S_2 is true

(D) Only S_1 is true

Key: (A)

Sol: S_1 is TRUE, With the particular focus on regular languages, the intended solution might have been something like using the pumping lemma to find x,y,z such that $xy^n z$ is in your language for every n , and then consider the subset $\{xy^n z | n \in A\}$, where A is your favorite undecidable set of natural numbers.

S_2 is also TRUE, Any finite language is regular.

31. Consider a Boolean function $f(w, x, y, z)$ such that

$$f(w, 0, 0, z) = 1$$

$$f(1, x, 1, z) = x + z$$

$$f(w, 1, y, z) = wz + y$$

The number of literals in the minimum sum-of-products expression of f is _____.

Key: (6)

Sol: It is given that for a function $f(w, x, y, z)$

$$f(w, 0, 0, z) = 1 \Rightarrow \text{i.e., when } x = 0, y = 0, f = 1$$

$f(w, x, 1, z) = x + z \Rightarrow$ when $w = 1, y = 1, f = x + z$

$f(w, 1, y, z) = wz + y \Rightarrow$ when $x = 1, f = wz + y$

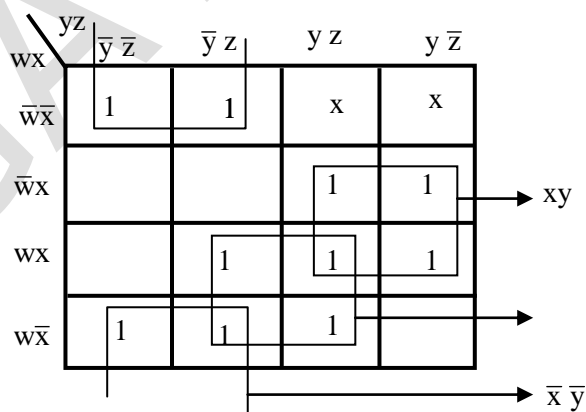
We need to obtain number of literal in minimum SOP of f.

w	x	y	z	x=0, y=0, f=1	w=1, y=1, f=x+z	x=1, f=wz+y	Overall f
0	0	0	0	1			1
0	0	0	1				
0	0	1	0				
0	0	1	1			0	0
0	1	0	1			0	1
0	1	1	0			1	1
0	1	1	1			1	1
0	1	1	1			0	1
1	0	0	0	1		0	1
1	0	0	1	1		0	1
1	0	1	0		0	0	0
1	0	1	1		1	0	1
1	1	0	0			0	0
1	1	0	1			1	1
1	1	1	0		1	1	1
1	1	1	1		1	1	1

No information for $w, x, y, z = 00010$ and 0011 , so f is taken don't care for unit combination

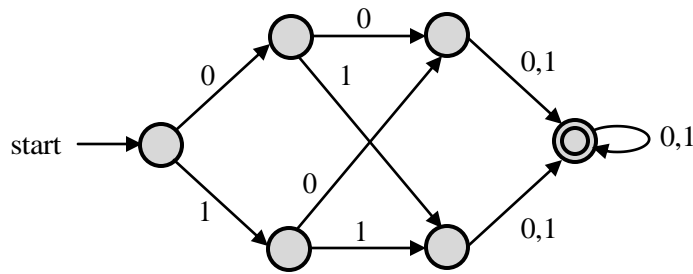
$f(w, x, yz) = \Sigma m(0,1,6,7,8,9,11,13,14,15) + d(2,3)$

Using k-map



So $f = \bar{x}\bar{y} + xy + wz$ (minimized SOP expression) each term of f containing 2 literal, so total 6 literal in expression of f.

32. Consider the following deterministic finite automaton (DFA).



The number of strings of length 8 accepted by the above automaton is _____.

Key: (256)

Sol: The regular expression for the above DFA is $(0(0+1)(0+1) + 1(0+1)(0+1))(0+1)^*$
 $(0+1)(0+1) (0+1)(0+1)^*$

Which can be defined as all string over $\{0,1\}$ of length greater than 3

For length 8, all 256 string will be accepted.

33. Let L_1 be a regular language and L_2 be a context-free language. Which of the following languages is/are context-free?

(A) $\overline{\overline{L_1} \cup \overline{L_2}}$

(B) $(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2)$

(C) $L_1 \cap \overline{L_2}$

(D) $L_1 \cap (L_2 \cup \overline{L_2})$

Key: (B,C)

Sol: $\overline{\overline{L_1} \cup \overline{L_2}} = L_1 \cap L_2$ is Intersection of Regular and context free and Outer set in Chomsky is preferred hence context free.

$L_1 \cap \overline{L_2}$

We know That Context free language is not closed under complement hence $\overline{L_2}$ may be the non-Context free language.

$L_1 \cap \overline{L_2}$ may be a **non-context free language**.

$L_1 \cup (L_2 \cup \overline{L_2})$ here $(L_2 \cup \overline{L_2})$ is Σ^* hence Regular. The union with regular language will make this as regular language hence context free

$(L_1 \cap L_2) \cap (\overline{L_1} \cup L_2)$, In this $(L_1 \cap L_2)$ is context free and $(\overline{L_1} \cup L_2)$ is also context free (L_1 is regular and its complement will be also regular).

The intersection of these two language may **not context free**, since Context free language are not closed under intersection operation

34. Consider a set-associative cache of size 2KB (1KB = 2^{10} bytes) with cache block size of 64 bytes. Assume that the cache is byte-addressable and a 32-bit address is used for accessing the cache. If the width of the tag field is 22 bits, the associativity of the cache is _____.

Key: (1)

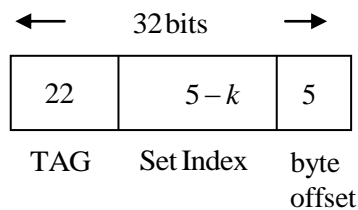
Sol: The block size is given as 64 (2^6) Bytes Byte offset will consists of 6 bits

The number of blocks are in the cache is $\frac{2\text{KB}}{2^6} = \frac{2^{11}\text{B}}{2^6\text{B}} = 2^5$

If the set is 2^k way set Associative then number of sets = $\frac{2^5}{2^k} = 2^{5-k}$

The number of bits required for the byte index us 5-k

Since main memory is addressed by 32 bits



$$22+5-k+5 = 32$$

$$K = 0;$$

The associativity us $2^0=1$

35. Consider the following ANSI C program.

```
# include <stdio.h>
int foo(int x, int y, int q)
{
    if((x <= 0) && (y <= 0))
        return q;
    if (x <= 0)
        return foo(x, y-q, q);
    if (y <= 0)
        return foo (x-q, y, q);
    return foo(x, y-q, q) + foo(x-q, y, q)
}
int main ( )
{
```

```
int r = foo(15, 15, 10);  
print ("%d", r);  
return 0;  
}
```

The output of the program upon execution is _____.

Key: (60)

36. For constants $a \geq 1$ and $b > 1$, consider the following recurrence defined on the non-negative integers:

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

Which one of the following options is correct about the recurrence $T(n)$?

- (A) If $f(n)$ is $\frac{n}{\log_2(n)}$, then $T(n)$ is $\Theta(\log_2(n))$
- (B) If $f(n)$ is $\Theta(n^{\log_b(a)})$, then $T(n)$ is $\Theta(n^{\log_b(a)})$
- (C) If $f(n)$ is $O(n^{\log_b(a)-\epsilon})$ for some $\epsilon > 0$, then $T(n)$ is $\Theta(n^{\log_b(a)})$
- (D) If $f(n)$ is $n \log_2(n)$, then $T(n)$ is $\Theta(n \log_2(n))$

Key: (C)

Sol: Master Theorem

The Master Theorem applies to recurrences of the following form:

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

Where $a \geq 1$ and $b > 1$ are constant and $f(n)$ is an asymptotically positive function.

There are 3 cases:

1. If $f(n) = O(n^{\log_b(a)-\epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b(a)})$.
2. If $f(n) = \Theta(n^{\log_b(a)} \log^k n)$ with $k \geq 0$, then $T(n) = \Theta(n^{\log_b(a)} \log^{k+1} n)$
3. If $f(n) = \Omega(n^{\log_b(a)+\epsilon})$ with $\epsilon > 0$, and $f(n)$ satisfies the regularity condition, then $T(n) = \Theta(f(n))$.

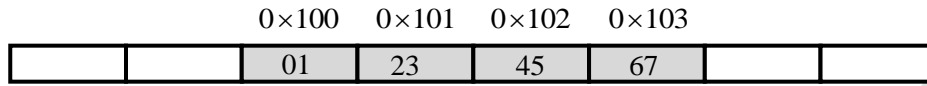
Regularity condition: $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n .

37. If the numerical value of a 2-byte unsigned integer on a little endian computer is 255 more than that on a big endian computer, which of the following choices represent(s) the unsigned integer on a little endian computer?

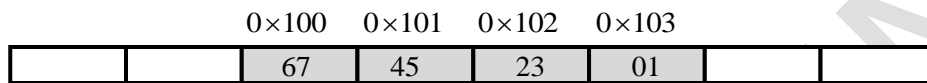
- (A) 0×0001 (B) 0×6665 (C) 0×0100 (D) 0×4243

Key: (C)

Sol:



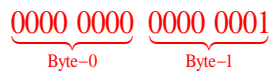
Big Endian



Little Endian

Suppose the number is 1 (A good guess)

In two byte unsigned representation



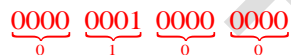
This representation is big endian representation.

The little-endian representation is



And which is 255 higher than bigger than little endian representation.

In hexadecimal representation



38. Suppose that $f: \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function on the interval $[-3, 3]$ and a differential function in the interval $(-3, 3)$ such that for every x in the interval, $f'(x) \leq 2$. If $f(-3) = 7$, then $f(3)$ is at most _____.

Key: (19)

Sol: By Lagrange's mean value theorem,

$$f'(x) = \frac{f(3) - f(-3)}{3 - (-3)} \text{ where } x \in (-3, 3) = \frac{f(3) - 7}{6}$$

$$\therefore f'(x) \leq 2$$

$$\therefore \frac{f(3) - 7}{6} \leq 2 \Rightarrow f(3) \leq 19$$

$$\therefore f(3) \text{ is at most } 19$$

39. For two n-dimensional real vectors P and Q, the operation $s(P, Q)$ is defined as follows:

$$s(P, Q) = \sum_{i=1}^n (P[i] \cdot Q[i])$$

Let L be a set of 10-dimensional non-zero real vectors such that for every pair of distinct vectors $P, Q \in L, s(P, Q) = 0$. What is the maximum cardinality possible for the set L?

- (A) 100 (B) 11 (C) 9 (D) 10

Key: (D)

Sol: $s(P, Q)$ represents Dot product of two vectors.

The dot product of $\mathbf{a} = \langle 1, 3, -2 \rangle$ and $\mathbf{b} = \langle -2, 4, -1 \rangle$ is

$$\mathbf{a} \cdot \mathbf{b} = 1(-2) + 3(4) + (-2)(-1) = 12$$

Two vectors are orthogonal if the angle between them is 90 degrees.

The dot product of two vectors is zero when they are perpendicular/orthogonal, as we are dealing with 10 dimensional vectors the maximum number of mutually-perpendicular vectors can be 10.

So option D.

40. Which of the following statement(s) is/are correct in the context of CPU scheduling?

- (A) The goal is to only maximize CPU utilization and minimize throughput
(B) Turnaround time includes waiting time
(C) Implementing preemptive scheduling needs hardware support
(D) Round-robin policy can be used even when the CPU time required by each of the processes is not known apriori.

Key: (B,C,D)

Sol: (B) Turnaround time includes waiting time : True

Turnaround time = waiting time + burst time

(C) Implementing preemptive scheduling needs hardware support : true

(D) Round-robin policy can be used even when the CPU time required by each of the processes is not known apriori. : True, SJF needs the CPU time required by each of the processes to be known apriori

41. Let S be a set consisting of 10 elements. The number of tuples of the form (A, B) such that A and B are subsets of S, and $A \subseteq B$ is _____.

Key: (59049)

Sol: The number of tuples = 3^{10} .

42. Consider the following ANSI C program

```
#include <stdio.h>
int main() {
    int arr[4][5]
    int i, j;
    for (i = 0; i < 4; i++){
        for (j = 0; j < 5; j++){
            arr[i][j] = 10*i+j;
        }
    }
    printf ("%d", *(arr[1] + 9));
    return 0;
}
```

What is the output of the above program?

- (A) 24 (B) 30 (C) 20 (D) 14

Key: (A)

Sol: The output of the above program will be 24.

43. Consider the string abbccddeee. Each letter in the string must be assigned a binary code satisfying properties:

1. For any two letters, the code assigned to one letter must not be a prefix of the code assigned to the other letter.
2. For any two letters of the same frequency, the letter which occurs earlier in the dictionary order is assigned a code whose length is at most the length of the code assigned to the other letter.

Among the set of all binary code assignments which satisfy the above two properties, what is the minimum length of the encoded string?

- (A) 25 (B) 23 (C) 21 (D) 30

Key: (B)

Sol: Using Huffman encoding, minimum length of encoded string =
 $1*3 + 2*3 + 2*2 + 2*2 + 3*2 = 21$

44. Consider a computer system with multiple shared resource types, with one instance per resource type. Each instance can be owned by only one process at a time. Owning and freeing of resources are done by holding a global lock (L). The following scheme is used to own a resource instance:

```
function OwnResource(Resource R)
  Acquire lock L// a global lock
  if R is available then
    Acquire R
    Release lock L
  else
    if R is owned by another process P then
      Terminate P, after releasing all resources owned by P
      Acquire R
      Restart P
      Release lock L
    end if
  end if
end function
```

Which of the following choice(s) about the above scheme is/are correct?

- (A) The scheme ensures that deadlocks will not occur
- (B) The scheme violates the mutual exclusion property
- (C) The scheme may lead to live-lock
- (D) The scheme may lead to starvation

Key: (A, C, D)

Sol: When R is owned by another process P then we can Terminate P, after releasing all resources owned by P.

So, consider a scenario where process R is free, P acquires lock, acquires R, releases lock, now process Q comes, acquires lock, since R is not available, so, Q Terminates P, after releasing all resources owned by P, then acquires R, restart P, release lock. So, now assume P comes and takes R away from Q. So, this continuous scenario can happen again and again..Thuslivelock is possible, so starvation is possible.

Deadlock is not possible, because no one is waiting. Process is terminating and restarting.

Mutual exclusion is satisfied because at one time, at most one process can own a resource.

45. Consider the following sets, where $n \geq 2$:

S_1 : Set of all $n \times n$ matrices with entries from the set {a, b, c}

S_2 : Set of all functions from the set $\{0, 1, 2, \dots, n^2 - 1\}$ to the set $\{0, 1, 2\}$

Which of the following choice(s) is/are correct?

- (A) There exists a surjection from S_1 to S_2
- (B) There does not exist an injection from S_1 to S_2
- (C) There does not exist a bijection from S_1 to S_2
- (D) There exists a bijection from S_1 to S_2

Key: (A, D)

Sol: There exist bijection from S_1 to S_2 as $|S_1| = |S_2| = 3^{n^2}$. And because bijection exists surjection will also exist.

46. Let $L \subseteq \{0,1\}^*$ be an arbitrary regular language accepted by a minimal DFA with k states. Which one of the following languages must necessarily be accepted by a minimal DFA with k states?

- (A) $L - \{01\}$
- (B) $L \cdot L$
- (C) $L \cup \{01\}$
- (D) $\{0,1\}^* - L$

Key: (D)

Sol: The Regular language (accepted by DFA) is closed under complementation and L and \bar{L} have same number of state in minimal DFA.

47. Suppose that P is a 4×5 matrix such that every solution of the equation $Px = 0$ is a scalar multiple of $[2 \ 5 \ 4 \ 3 \ 1]^T$. The rank of P is _____

Key: (4)

Sol: $PX = 0$ is homogeneous system, where X (is solution) is a scalar multiple of $[2 \ 5 \ 4 \ 3 \ 1]^T$
 $\Rightarrow PX = 0$ has infinitely many solutions and all solutions are of the form $\lambda \cdot X$, where λ is scalar
 \Rightarrow number linearly independent solutions is 1
 $\Rightarrow n - r = 1$, where n is number of unknowns and
 r is rank of P . i.e., $r = n - 1 = 5 - 1 = 4$
 \therefore Rank of P is 4

48. Consider a network using the pure ALOHA medium access control protocol, where each frame is of length 1,000 bits. The channel transmission rate is 1 Mbps ($= 10^6$ bits per second). The aggregate number of transmission across all the nodes (including new frame transmissions and retransmitted frames due to collisions) is modelled as a Poisson process with a rate of 1,000 frames per second. Throughput is defined as the average number of frames successfully transmitted per second. The throughput of the network (rounded to the nearest integer) is _____.

Key: (135)

Sol: Transmission time = $1000 / 10^6 = 1 \text{ ms}$

Efficiency of pure aloha = $G * e^{-2G}$

Here $G = 1$,

Efficiency = $1 * 2.718^{-2*1} = 0.1353$

Throughput = $1000 * 0.1353 = 135.3 = 135$ (rounded to nearest integer)

49. The format of the single-precision floating-point representation of a real number as per the IEEE 754 standard is as follows

sign	exponent	mantissa
------	----------	----------

Which one of the following choices is correct with respect to the smallest normalized positive number represented using the standard?

- (A) exponent = 00000000 and mantissa = 000000000000000000000000
- (B) exponent = 00000001 and mantissa = 0000000000000000000000001
- (C) exponent = 00000000 and mantissa = 0000000000000000000000001
- (D) exponent = 00000001 and mantissa = 0000000000000000000000000

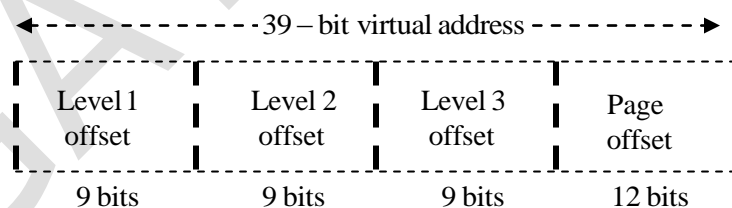
Key: (D)

Sol: For the smallest positive normalized number

Biased exponent will be equal to 00000001, since 00000000 is reserved for the special purpose.

And normalized mantissa can be $1. \underbrace{000\ 0000\ 0000\ 0000\ 0000\ 0000}_{23\ \text{zeros}}$

50. Consider a three-level page table to translate a 39-bit virtual address to a physical address as shown below.



The pages size is 4KB ($1\text{KB} = 2^{10}$ bytes) and page table entry size at every level is 8 bytes. A process P is currently using 2GB ($1\text{GB} = 2^{30}$ bytes) virtual memory which is mapped to 2GB of physical memory. The minimum amount of memory required for the page table of P across all levels is _____KB.

Key: (12)

Sol: As in the question minimum memory is asked so we will consider 1 page at each level. So page table size at one level = $2^9 * 8 = 4\text{KB}$

Total page table size = $4\text{KB} * 3 = 12\text{KB}$

51. Consider the following ANSI C code segment:

```

z = x + 3 + y ->f1 + y->f2;
for (i = 0; i < 200; i = i + 2){
    if (z > i) {
        p = p + x + 3;
        q = q + y ->f1
    } else {
        p = p + y ->f2;
        q = q + x + 3;
    }
}

```

Assume that the variable y points to a struct (allocated on the heap) containing two fields f1 and f2, and the local variables x, y, z, p, q and i are allotted registers. Common sub-expression elimination (CSE) optimization is applied on the code. The number of addition and dereference operations (of the form y->f1 or y->f2) in the optimized code, respectively, are:

- (A) 403 and 102 (B) 303 and 102 (C) 203 and 2 (D) 303 and 2

Key: (B)

Sol: The Optimization code will be

<pre> t1= x+3; t2 = y->f1; t3= y->f2 z = t1 + t2 + t3; for (i = 0; i < 200; i = i + 2) { if (z > i) { p = p + t1; q = q + t2; } else { p = p + t3; q = q + t1; } } </pre>	<p>1 Addition</p> <p>1 reference</p> <p>2 reference</p> <p>2 Addition</p> <p>100 Addition</p> <p>Either if or else part will be execute ,</p> <p>If Part has 2 addition</p> <p>Else part has 2 addition</p> <p>Total 200 addition</p>
Total	303 Addition and 2 references

52. For a given biased coin, the probability that the outcome of a toss is a head is 0.4. This coin is tossed 1,000 times. Let X denote the random variable whose value is the number of times that head appeared in these 1,000 tosses. The standard deviation of X (rounded to 2 decimal place) is _____.

Key: (15.49)

Sol: Let $p = P_r(\text{getting a head in any toss}) = 0.4 \Rightarrow p$ is not very small

$$\Rightarrow q = 1 - p = 0.6$$

$n = 1000$ (very large)

Let X be a random variable denote number of times that head appears in 1000 tosses.

$\Rightarrow X$ is a Binomial distribution

$$\therefore \text{Standard deviation is } \sqrt{npq} = \sqrt{240} \approx 15.49$$

53. Consider the cyclic redundancy check (CRC) based error detecting scheme having the generator polynomial $X^3 + X + 1$. Suppose the message $m_4m_3m_2m_1m_0 = 11000$ is to be transmitted. Check bits $c_2c_1c_0$ are appended at the end of the message by the transmitter using the above CRC scheme. The transmitted bit string is denoted by $m_4m_3m_2m_1m_0c_2c_1c_0$. The value of the checkbit sequence $c_2c_1c_0$ is

(A) 101

(B) 111

(C) 100

(D) 110

Key: (C)

Sol: $x^3 + x + 1 = 1011$

$$\begin{array}{r}
 1011 \overline{) 11000000} \\
 \underline{1011} \\
 1110 \\
 \underline{1011} \\
 1010 \\
 \underline{1011} \\
 100
 \end{array}$$

54. Consider the following ANSI C function:

```
Int SomeFunction (int x, int y )
{
    if ((x == 1) || (y == 1)) return 1;
    if (x == y) return x;
    if (x > y) return SomeFunction(x -y , y);
    if (y > x) return SomeFunction(x, y - x);
}
```

The value returned by SomeFunction (15, 255) is _____.

Key: (15)

55. Suppose the following functional dependencies hold on a relation U with attributes P, Q, R, S and T:

$P \rightarrow QR$

$RS \rightarrow T$

Which of the following functional dependencies can be inferred from the above functional dependencies?

(A) $P \rightarrow R$

(B) $PS \rightarrow T$

(C) $PS \rightarrow Q$

(D) $R \rightarrow T$

Key: (A,B,C)

Sol: $(P)^+ = \{P, Q, R\}$

$(PS)^+ = \{P, QR, S, T\}$

$(R)^+ = \{R\}$

