GENERAL APTITUDE

1. Let X be a continuous random variable denoting the temperature measured. The range of temperature is [0, 100] degree Celsius and let the probability density function of X be f(x) = 0.01 for $0 \le X \le 100$.

The mean of X is _______.

(A) 25.0 (B) 5.0 (C) 50.0 (D) 2.5

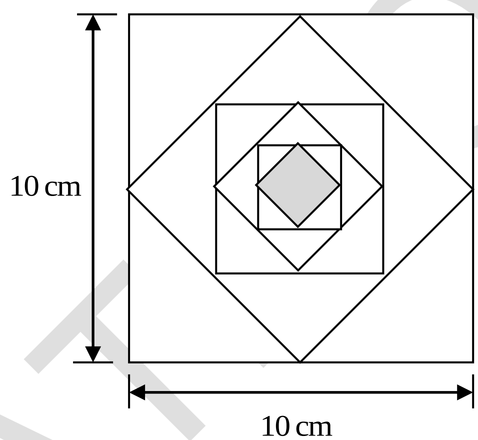
Key: **(C)**

Sol: $E(X) = \text{Mean of } X$

$$= \int_0^{100} x.f(x)\,dx$$

$$= \int_0^{100} x.(0.01)\,dx = 0.01\left(\frac{x^2}{2}\right)_0^{100} = \frac{0.01}{2}(100)^2 = \frac{1}{200} \times 100 \times 100 = 50$$

2.



In the figure shown above, each inside square is formed by joining the midpoints of the sides of the next larger square. The area of the smallest square (shaded) as shown, in cm^2 is:

(A) 6.25 (B) 3.125 (C) 1.5625 (D) 12.50

Key: **(B)**

Sol: Using Pythagoras theorem, in each square, we can find side length.

$\therefore$ Side of smallest square becomes $= \frac{5}{\sqrt{8}}$

$\therefore$ Area of smallest square $= \frac{5}{\sqrt{8}} \times \frac{5}{\sqrt{8}} = \frac{25}{8} = 3.125$

3. The important of sleep is often overlooked by students when they are preparing for exams. Research has consistently shown that sleep deprivation greatly reduces the ability to recall the material learnt. Hence, cutting down on sleep to study longer hours can be counterproductive.

Which one of the following statements is the CORRECT inference from the above passage?

(A) To do well in exam, adequate sleep must be part of the preparation

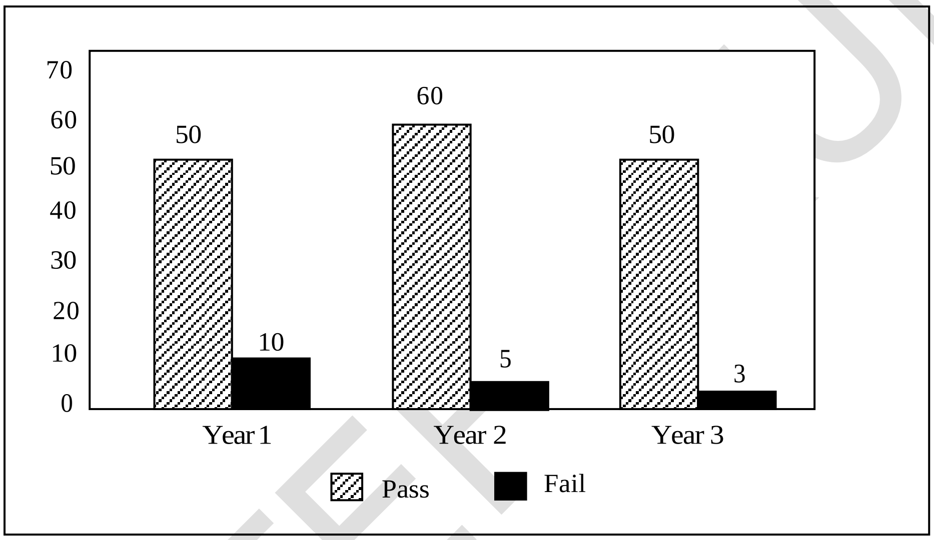
(B) Students are efficient and are not wrong in thinking that sleep is a waste of time

(C) If a student is extremely well prepared for an exam, he needs little or no sleep

(D) Sleeping well alone is enough to prepare for an exam. Studying has lesser benefit

Key: (A)

4.



The number of students passing or failing in an exam for a particular subject is presented in the bar chart above. Students who pass the exam cannot appear for the exam again. Students who fail the exam in the first attempt must appear for the exam in the following year. Students always pass the exam in their second attempt.

The number of students who took the exam for the first time in the year 2 and the year 3 respectively, are________.

(A) 65 and 53 (B) 55 and 53 (C) 60 and 50 (D) 55 and 48

Key: (D)

Sol: From the bar graph

In year 2: Total number of students who took the exam = 60 + 5 = 65

But who took the exam for 1st time = 65 – 10 = 55.

In year 3: Total number of students who took the exam = 50 + 3 = 53

But who took the exam for 1st exam = 53 –5 = 48

Hence option (D) is correct.

5. Which one of the following numbers is exactly divisible by $\left(11^{13}+1\right)$?

(A) $11^{52}-1$ (B) $11^{39}-1$ (C) $11^{33}+1$ (D) $11^{26}+1$

Key: (A)

Sol: $11^{52}=\left(11^{26}\right)^{2}-1^{2}$

$=\left(11^{26}+1\right)\left(11^{26}-1\right)$

$=\left(11^{26}+1\right)\left[\left(11^{13}\right)^{2}-\left(1^{2}\right)\right]$

$=\left(11^{26}+1\right)\left[\left(11^{13}\right)-1\right]\left(11^{13}+1\right)$

$\therefore 11^{13}+1$ is divisor of $11^{52}-1$.

6. Seven cars P, Q, R, S, T, U and V are parked in a row not necessarily in that order. The cars T and U should be parked next to each other. The cars S and V also should be parked next to each other, whereas P and Q cannot be parked next to each other. Q and S must be parked next to each other. R isparked to the immediate right of V. T is parked to the left of U.

Based on the above statements, the only INCORRECT option given below is:

(A) There are two cars parked in between Q and V

(B) Q and R are not parked together

(C) V is the only car parked in between S and R

(D) Car P is parked at the extreme end

Key: (A)

Sol: According to the given information, we can arrange:

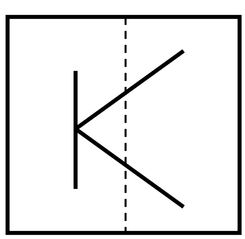
Q S V R T U P

Statements: (B), (C), (D) are true and

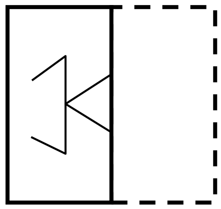
Statements (A) is incorrect.

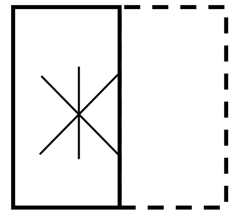
($\because$ There is only one car between Q and V)

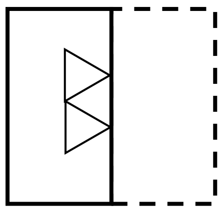
7.

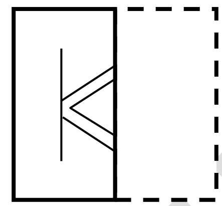


A transparent square sheet show above is folded along the dotted line. The folded sheet will look like

(A) 

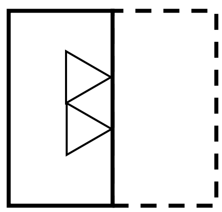
(B) 

(C) 

(D) 

Key: (C)

Sol: If the square sheet is folded along the dotted line, then the folded sheet will look like as follows.



8. The people ______________ were at the demonstration were from all sections of society.

(A) whose (B) which (C) whom (D) who

Key: (D)

9. Oasis is to sand as island is to ____________.

Which one of the following options maintains a similar logical relation in the above sentence?

(A) Water (B) Mountain (C) Land (D) Stone

Key: (A)

10. For a regular polygon having 10 sides, the interior angle between the sides of the polygon, in degrees, is:

(A) 396 (B) 144 (C) 216 (D) 324

Key: (B)

Sol: The formula for calculating the sum of interior angles in a regular polygon is $(n-2) \times 180°$.

Here $n = 10$

$\therefore$ The sum of interior angles of a decagon $= (10-2) \times 180 = 8 \times 180°$.

$\therefore$ Each interior angle $= \dfrac{8 \times 180°}{10} = 144°$.

ELECTRICAL ENGINEERING

1. In a single-phase transformer, the total iron loss is 2500 W at nominal voltage of 440V and frequency 50Hz. The total iron loss is 850W at 220V and 25 Hz. Then, at nominal voltage and frequency, the hysteresis loss and eddy current loss respectively are

(A) 900 W and 1600 W
(B) 250 W and 600 W
(C) 1600 W and 900 W
(D) 600 W and 250 W

Key: (A)

Sol: Given:

$W_1 = 2500W$

$V_1 = 440V$

$f_1 = 50Hz$

$W_2 = 850W$

$V_2 = 220V$

$f_2 = 25Hz$

Iron loss, $W_i = W_h + W_f$ ...(A)

Hysteresis loss, $W_h = A_f$ ...(i)

$W_f = Bf^2$ ...(ii)

Substitute (i) and (ii) in (A) equation, we get

$W_i = Af + Bf^2$

$\frac{W_i}{f} = A + Bf$ ...(B)

Substitute, $f_1 = 50Hz$ in (B)

$\frac{2500}{50} = A + 50B$

$50 = A + 50B$ ...(1)

Substitute $f_2 = 25Hz$ in (B)

$\frac{850}{25} = A + 25B$

$34 = A + 25B$ ...(2)

Solve (1) and (2) equations, we get

A = 18, B = 0.64

Substitute A and B values in equation (i) and (ii) to find W_h and W_e.

$$W_h = A \times f = 18 \times 50 = 900W$$

$$W_e = Fyf = 0.64 \times 50^2 = 1600W$$

$$\frac{V_2}{f_2} = \frac{V_1}{f_1}$$

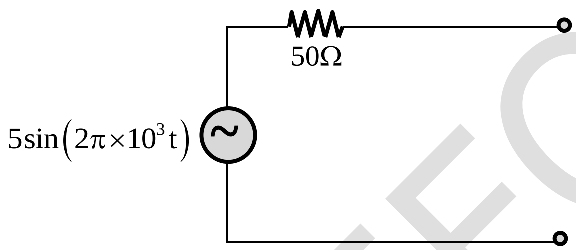
$$\frac{220}{25} = \frac{440}{50}$$

So, $B_{m2} = B_{m1}$

2. A signal generator having a source resistance of $50\,\Omega$ is set to generate a 1 kHz sinewave. Open circuit terminal voltage is 10V peak-to-peak. Connecting a capacitor across the terminals reduces the voltage to 8V peak-to-peak. The value of this capacitor is ___________ μF. (Round off to 2 decimal places).

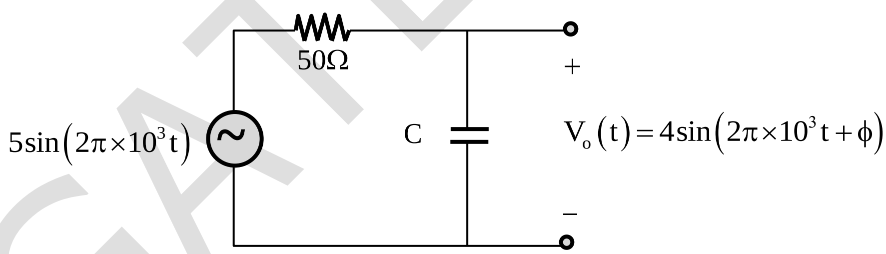
Key: **(2.38)**

Sol: As per the given information, before connecting the capacitor the circuit looks like



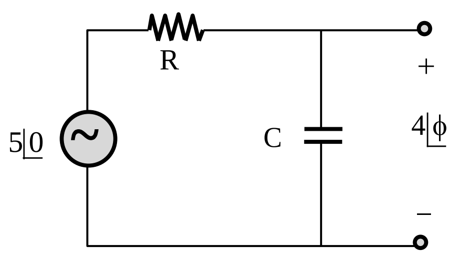
If peak amplitude is 5 then peak to peak amplitude is 10.

After connecting capacitor at load, the circuit should look like



If peak amplitude is 4 then peak to peak amplitude 8.

Here out interest is only amplitude not phase



$$V_0(j\omega)=\frac{1/j\omega C}{R+1/j\omega C}V_i(j\omega)=\frac{1}{1+j\omega RC}5\underline{|0}$$

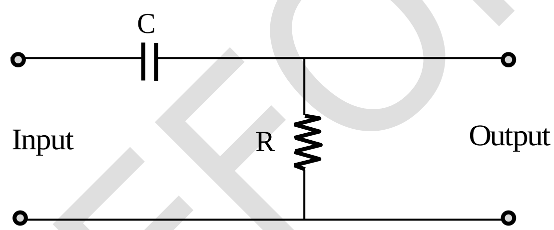
$$\Rightarrow |V_0(j\omega)|=\frac{5}{\sqrt{1+(\omega RC)^2}}$$

$$\Rightarrow 4=\frac{5}{\sqrt{1+(\omega RC)^2}}$$

$$\Rightarrow 1+(\omega RC)^2=\left(\frac{5}{4}\right)^2$$

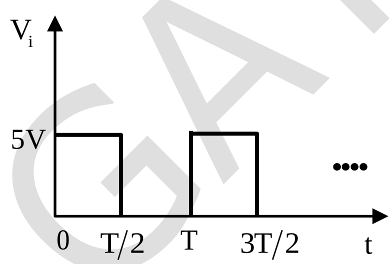
$$\Rightarrow C=\sqrt{\frac{(1.25)^2-1}{\omega^2R^2}}=\sqrt{\frac{(1.25)^2-1}{(2\pi\times10^3)^2\times(50)^2}}=\sqrt{\frac{0.5625}{9.86\times10^{10}}}=2.38\mu F$$

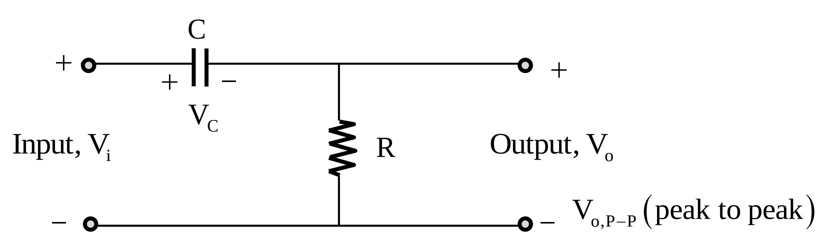
3. A 100 Hz square wave, switching between 0V and 5V, is applied to a CR high-pass filter circuit as shown. The output voltage waveform across the resistor is 6.2V peak-to-peak. If the resistance R is 820 Ω, then the value C is ___________ μF. (Rounded off to 2 decimal places).



Key: **(12.43)**

Sol: Given,





From KVL, $V_i-V_C-V_0=0\Rightarrow V_0=V_i-V_C$

(Charging)

For $0<t<\frac{T}{2}$, $V_C(0)=V_{C,min}$ (assumed)

$V_C(\infty) = 5V$ ($\because$ input available is maximum 5V capacitor can change maximum upto 5 only)

As $V_C(t) = V(\infty) + [V(0) - V(\infty)]e^{t/\tau}$

$\therefore V_C(t) = 5 + (V_{C,min} - 5)e^{-t/\tau}$

At $t = \frac{T}{2}, V_C = V_{C,max} \Rightarrow V_{C,max} = 5 + (V_{C,min} - 5)e^{-T/2\tau} \quad \ldots(1)$

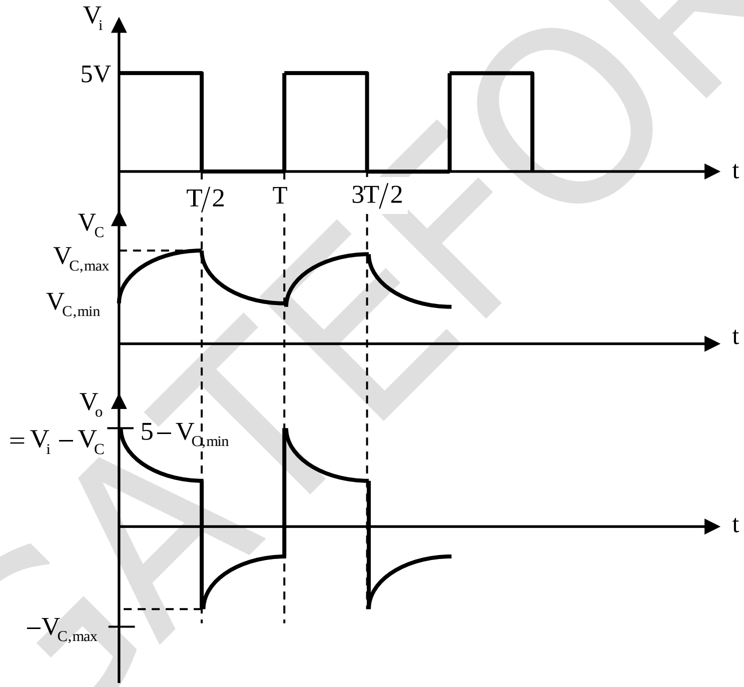
(Discharging)

For $\frac{T}{2} < t < T,\quad V_C(0) = V_{C,max},\ V_1(\infty) = 0$

$\therefore \quad V_C = 0 + (V_{C,max} - 0)e^{-\frac{(t-T/2)}{\tau}} = V_{C,max}e^{-\frac{(t-T/2)}{\tau}}$

At $t = T,\ V_C = V_{C,min} \Rightarrow V_{C,min} = V_{C,max}e^{-T/2\tau} \quad \ldots(ii)$

From above analysis the corresponding waveforms are,



Given, $V_{o,P-P} = 6.2V$

$\Rightarrow (5 - V_{C,min}) - (-V_{C,max}) = 6.2$

$\Rightarrow V_{C,max} - V_{C,min} = 1.2V \quad \ldots(iii)$

Solving (i), (ii) and (iii), $\tau = 0.0102$ using $T = \frac{1}{100}$ sec

$\Rightarrow RC = 0.0102 \Rightarrow C = 12.43\,\mu F$

4. Let f(t) be an even function, i.e., $f(-t)=f(t)$ for all t. Let the Fourier transform of f(t) be defined as $F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$. Suppose $\frac{dF(\omega)}{d\omega}=-\omega F(\omega)$ for all ω, and $F(0)=1$. Then

(A) $f(0)>1$ (B) $f(0)=0$ (C) $f(0)<1$ (D) $f(0)=1$

Key: (C)

Sol: It is given that $f(-t)=f(t)=f(t), \forall t$

$$\frac{dF(\omega)}{d\omega}=-\omega F(\omega), \forall \omega$$

$$F(0)=1$$

$$\frac{dF(\omega)}{F(\omega)}=-\omega d\omega$$

$$\Rightarrow \ell nF(\omega)=\frac{-\omega^2}{2}+C$$

$$\Rightarrow F(\omega)=e^{-\frac{\omega^2}{2}+C}$$

$$\Rightarrow F(0)=e^{0+C}$$

$$\Rightarrow 1=e^{C}$$

$$\Rightarrow C=0$$

$$\text{So } F(\omega)=e^{-\frac{\omega^2}{2}}$$

We know

$$e^{-at^2}\leftrightarrow\sqrt{\frac{\pi}{a}}e^{\frac{-\omega^2}{4a}}$$

If $a=\frac{1}{2}$ then $e^{-\frac{t_2}{2}}\leftrightarrow\sqrt{2\pi}\, e^{-\frac{\omega^2}{2}}$

$$e^{-\frac{\omega^2}{2}}\leftrightarrow\frac{1}{\sqrt{2\pi}}e^{-t^2/2}$$

So, $f(t)=\frac{1}{\sqrt{2\pi}}e^{-\frac{t_2}{2}}$

$$f(0)=\frac{1}{\sqrt{2\pi}} \text{ so } f(0)<1$$

5. Consider the table given:

Constructional feature	Machine type	Mitigation
(P) Damper bars	**(S)** Induction motor	**(X)** Hunting
(Q) Skewed rotor slots	**(T)** Transformer	**(Y)** Magnetic locking
(R) Compensating winding	**(U)** Synchronous machine	**(Z)** Armature reaction
	(V) DC machine	

The correct combination that relates the constructional feature, machine type and mitigation is

(A) P-V-X, Q-U-Z, R-T-Y
(B) P-T-Y, Q-V-Z, R-S-X
(C) P-U-X, Q-V-Y, R-T-Z
(D) P-U-X, Q-S-Y, R-V-Z

Key: (D)

Sol: **(i)** Synchronous machines are not self-starting machines. These machines are made self-starting by providing a special winding in the rotor poles known as damper winding. When there is change in load, excitation or change in other conditions of the systems rotor of the synchronous motor will oscillate to and fro about an equilibrium position. These oscillationsbecomes more violent and resulting in loss of synchronism of the motor and comes to halt.

(ii) If the rotor and the stator conductors are parallel to each other, there is a strong possibility of the magnetic locking between the rotor and stator. Therefore, the rotor slots are skewed.

(iii) In order to neutralize the cross-magnetizing effect of armature reaction, a compensating winding is used in DC machines. The compensating windings consist of a series coils embedded in slots in the pole faces. These coils are connected in series with the armature.

Hence, PUX – QSY – RVZ'

6. Inductance is measured by

(A) Schering bridge
(B) Kelvin bridge
(C) Wien bridge
(D) Maxwell bridge

Key: (D)

Sol: For inductance measurement Maxwell's bridge is used.

- Schering bridge is used for measurement of capacitance.
- Wien bridge is used for measurement of frequency.
- Kelvin bridge is used for measurement of resistance.

7. Suppose I_A, I_B and I_C are a set of unbalanced current phasors in a three-phase system. The phase-B zero-sequence current $I_{B0} = 0.1\angle 0°$ p.u. If phase-A current $I_A = 1.1\angle 0°$ p.u and phase-C current $I_C = (1\angle 120° + 0.1)$p.u. then I_B in p.uis

(A) $1\angle -120° + 0.1\angle 0°$
(B) $1\angle -240° + 0.1\angle 0°$
(C) $1.1\angle 240° - 0.1\angle 0°$
(D) $1.1\angle -120° + 0.1\angle 0°$

Key: (A)

Sol: Given, $I_{B0} = 0.1\angle 0$ pu

$I_A = 1.1\angle 0$ pu

$I_C = (1\angle 120° + 0.1)$ pu

We know, $\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix}$ and $I_{A0} = I_{B0} = I_{C0}$

$\therefore \quad I_A = I_{A0} + I_{A1} + I_{A2} = 1.1\angle 0$

$\Rightarrow I_{A1} + I_{A2} = 1.1\angle 0 - I_{A0} = 1.0 \qquad \ldots(i)$

Also, $I_C = I_{A0} + \alpha I_{A1} + \alpha^2 I_{A2} = 1\angle 120 + 0.1$

$\Rightarrow \alpha I_{A1} + \alpha^2 + I_{A2} = 1\angle 120° \qquad \ldots(ii)$

Solving (i) and (ii),

$I_{A1} = 1.0$ pu and $I_{A2} = 0$

Then, $I_B = I_{A0} + \alpha^2 I_{A1} + \alpha I_{A2}$

$= I_{B0} + \alpha^2 I_{A1} + \alpha I_{A2}$

$= 0.1 + 1\angle 240 + 0$

$= 1\angle -120 + 0.1\angle 0$

8. An alternator with internal voltage of $1\angle\delta_1$ p.u and synchronous reactance of 0.4 p.u is connected by a transmission line of reactance 0.1 p.u to a synchronous motor having synchronous reactance 0.35 p.u and internal voltage of $0.85\angle\delta_2$ p.u. If the real power supplied by the alternator is 0.866 p.u, then $(\delta_1 - \delta_2)$ is ___________ degrees. (Round off to 2 decimal places.)

(Machines are of non-salient type. Neglect resistances).

Key: (60)

Sol: $E = 1\lfloor\delta V$

$X_S = j0.4$ pu

$R_a = j0.1$ pu

$X_{S1} = j0.35$ pu

$P = 0.866$ pu

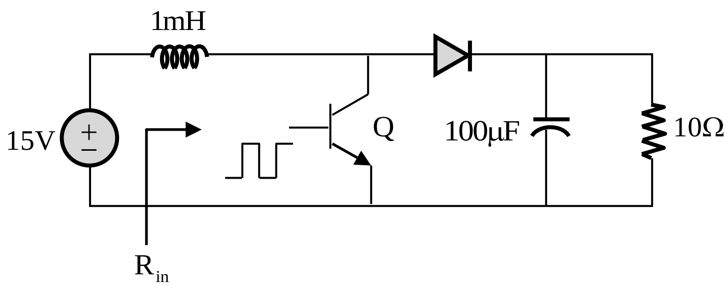
Total power transferred,

$$P = \frac{E_f V}{X_{eq}} \sin(\delta_1 - \delta_2) \Rightarrow 0.866 = \frac{1 \times 0.85}{0.1 + 0.35 + 0.4} \sin(\delta_1 - \delta_2)$$

$\sin(\delta_1 - \delta_2) = 0.866$

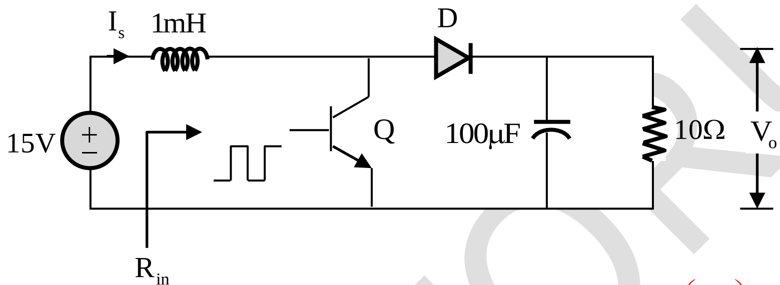
$\delta_1 - \delta_2 = 60°$

9. Consider the boost converter shown. Switch Q is operating at 25 kHz with a duty cycle of 0.6. Assume the diode and switch to be ideal. Under steady-state condition, the average resistance R_{in} as seen by the source is ___________Ω (Round off to 2 decimal places).



Key: **(1.6)**

Sol: Given,



$f_{operating} = 25\text{kHz and } D = 0.6$ We need to calculate average $R_{in} = \left(\frac{V_S}{I_S}\right)_{DC}$

$$V_o = \frac{V_S}{1-D} = \frac{15}{1-0.6} = 37.5\text{V}, \quad I_o = \frac{V_o}{R_o} = \frac{37.5}{10} = 8.75\text{A}$$

$$I_S = \frac{I_o}{1-D} = \frac{3.75}{0.4} = 9.375\text{A}$$

$$\boxed{R_{in} = \frac{15}{9.375} = 1.6\Omega}$$

Hence, the average resistance R_{in} as seen by the source is 1.6Ω.

10. If the input x(t) and output y(t) of a system are related as $y(t) = \max(0, x(t))$, then the system is

(A) non-linear and time-invariant

(B) linear and time-invariant

(C) non-linear and time-variant

(D) linear and time-variant

Key: **(A)**

Sol: It is given that $y(t) = \text{Max}(0, x(t))$

$$y_1(t) = \text{Max}\left[0, x_1(t)\right]$$
$$y_2(t) = \text{Max}\left[0, x_2(t)\right]$$

$y_3(t) = \text{Max}\left[0, \left(x_1(t) + x_2(t)\right)\right]$

$y_4(t) = \text{Max}\left[0, \alpha x(t)\right]$

$y_5(t) = \alpha \text{Max}\left[0, x(t)\right]$

For additively we want $y_3(t) = y_1(t) + y_2(t)$

Homogeneity we want $y_4(t) = y_5(t)$

Let $x(1) = 1$ and $x_2(t) = -1$ (both D.C)

Then $y_1(t) = \text{Max}(0,1) = 1$

$y_2(t) = \text{Max}(0,-1) = 0$

$y_3(t) = \text{Max}(0,0) = 0$

$y_3(t) \neq y_1(t) + y_2(t)$ So, system is nonlinear.

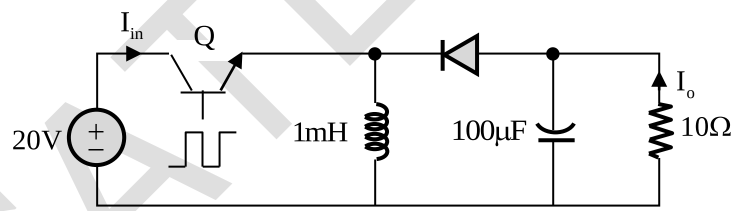
If input is $x(t - t_o)$ then $y_1(t) = \text{Max}\left[0, x(t - t_o)\right]$

$y(t - t_o) = \text{Max}\left[0, x(t - t_o)\right]$

Since $y_1(t) = y(t_1 - t_o)$ system is time invariant

So, system is nonlinear and time invariant.

11. Consider the buck-boost converter shown. Switch Q is operating at 25 kHz and 0.75 duty-cycle. Assume diode and switch to be ideal. Under steady-state condition, the average current flowing through the inductor is ____________A.



Key: **(24)**

Sol: Given, $f_{operating} = 25\,\text{kHz}, D = 0.75$ and converter is buck boost.

First of all we should check for continuity or discontinuity of converter

$$L_C = \frac{(1-D)^2 R}{2f} = \frac{(1-0.75)^2 \times 10}{2 \times 25 \times 10^3} = 1.25 \times 10^{-5} \text{ or } 12.5\,\mu\text{H}$$

$L(\text{given}) > L_C$ hence it is continuous mode of operation.

$$I_o = \frac{V_o}{R} = \frac{\frac{DV_{sc}}{1-D}}{R} = 6A$$

$$\therefore I_L = \frac{I_D}{1-D} = \frac{6}{1-0.75} = 24A$$

Hence, the average current flowing through the inductor is 24A.

12. Let A be a 10 × 10 matrix such that A^5 is a null matrix, and let I be the 10 × 10 identity matrix. The determinant of A + I is ____________.

Key: **(1)**

Sol: Given $A^5 = 0$

$\Rightarrow$ A is Nilpotent matrix

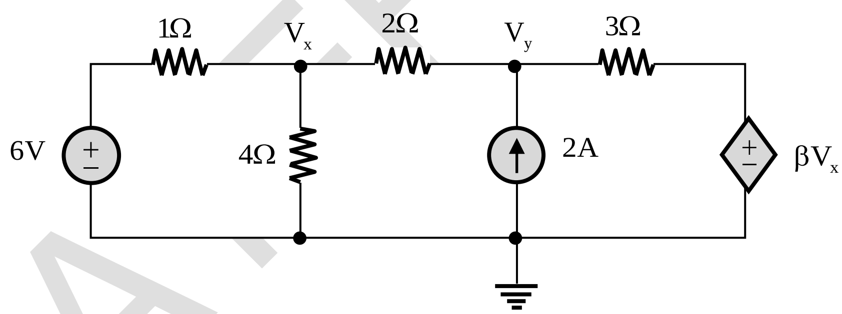
$\Rightarrow$ eigen values of A are all '0'

$\Rightarrow$ eigen values of the matrix A + I are all equal to 0+1 = 1

$\therefore$ Determinant of (A+ I) is product of eigen values of (A+ I)

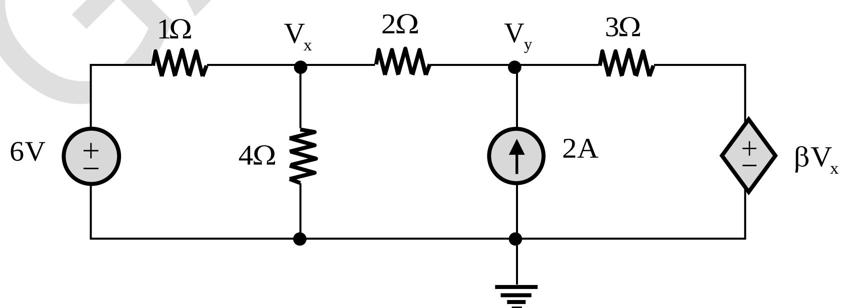
= 1 × 1× 1… ×1 (10 times) =1 (Using properties of eigen values)

13. In the given circuit, for voltage V_y to be zero, the value of β should be (Round off to 2 decimal places).



Key: **(–3.25)**

Sol: In the following network we need to obtain value of β for which V_y is 0



Writing Nodal equation at node X we have

$$\frac{V_x-6}{1}+\frac{V_x-V_y}{2}+\frac{V_x}{4}=0$$

$$\Rightarrow 4V_x-24+2V_x-2V_y+V_x=0$$

$$\Rightarrow 7V_x-2V_y=24$$

If $V_y=0\Rightarrow V_x=\frac{24}{7}V$

Writing nodal equation at node y we have

$$\frac{V_y-V_x}{2}+\frac{V_y-\beta V_x}{3}-2=0$$

If $V_y=0$ then $\frac{-V_x}{2}-\frac{\beta V_x}{3}=2$

$$\Rightarrow -3V_x-2\beta V_x=12$$

$$\Rightarrow V_x(3+2\beta)=-12$$

$$\Rightarrow 3+2\beta=\frac{-12}{V_x}\Rightarrow 2\beta=\frac{-12}{V_x}=3$$

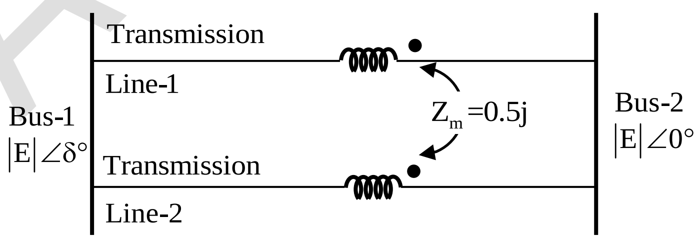
$$\Rightarrow 2\beta=\left(-12\times\frac{7}{24}\right)-3$$

$$\Rightarrow 2\beta=-3.5-3$$

$$\Rightarrow \beta=\frac{-6.5}{2}=-3.25$$

$$\Rightarrow \beta=-3.25$$

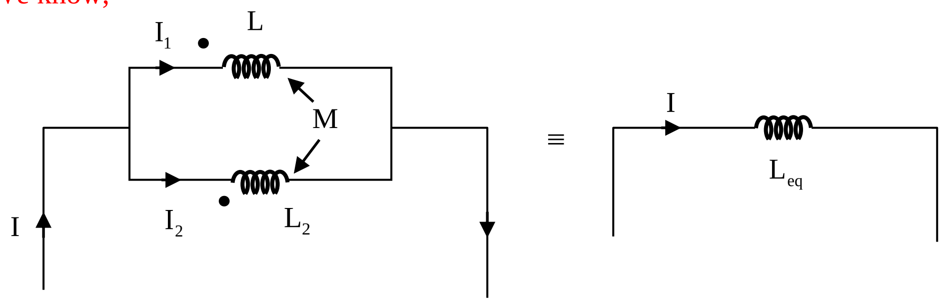
14. In the figure shown, self-impedance of the two transmission lines are 1.5j p.u each, and $Z_m=0.5j$ p.u is the mutual impedance. Bus voltage shown in the figure are in p.u. Given that $\delta>0$, the maximum steady-state real power that can be transferred in p.u from Bus-1 to Bus-2 is



(A) $2|E||V|$ (B) $\frac{3|E||V|}{2}$ (C) $|E||V|$ (D) $\frac{|E||V|}{2}$

Key: (C)

Sol: We know,



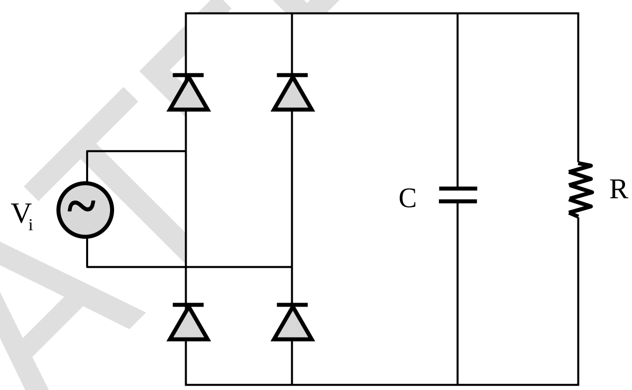
Where, $L_{eq} = \dfrac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$

Or, $j\omega L_{eq} = \dfrac{(j\omega L_1)(j\omega L_2) - (j\omega M)^2}{(j\omega L_1) + (j\omega L_2) - 2(j\omega M)}$

Or, $X_{eq} = \dfrac{X_{1s}X_{2s} - X_m^2}{X_{1s} + X_{2s} - 2X_m} = \dfrac{1.5\times 1.5 - 0.5^2}{1.5 + 1.5 - 2\times 0.5} = 1\,\text{pu}$

$\therefore P_{max} = \dfrac{|E||V|}{X_{eq}} = \dfrac{|E|\times|V|}{1} = |E||V|$

15. In the circuit shown, the input V_i is a sinusoidal AC voltage having and RMS value of $230V \pm 20\%$. The worst-case peak-inverse voltage seen across any diode is __________V. (Round off to 2 decimal places).



Key: **(390.32)**

Sol: Given, $V_i = 230 \pm 20\%$, and as given diagram

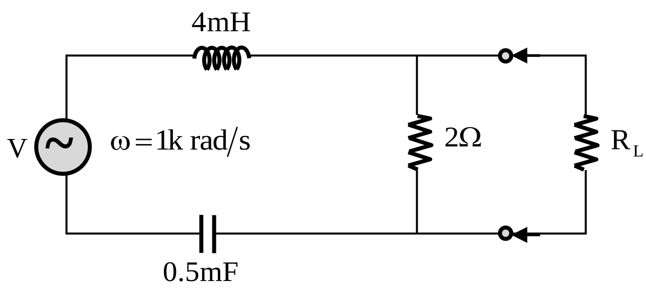
We have to calculated worst case peak inverse voltage seen across any diode.

$$\left(V_{D\max}\right)_{PIV} = \sqrt{\left(V_i\right)^2_{\max} + \left(V_{C\max}\right)^2} = \sqrt{\left(\frac{230\times 1.2\times\sqrt{2}}{\sqrt{2}}\right)^2 + \left(230\times 1.2\right)^2} = \sqrt{2\times\left(276\right)^2}$$

$$\boxed{\left(V_{D\max}\right)_{PIV} = 390.32V}$$

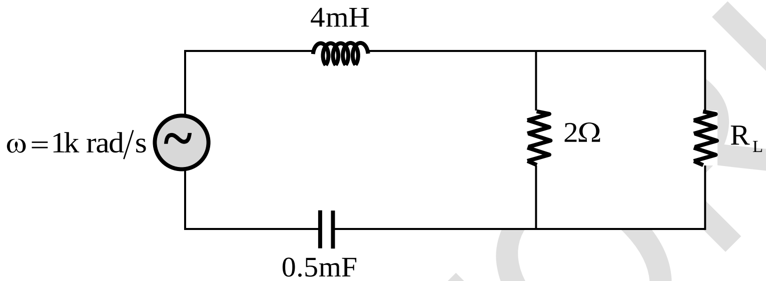
The worst case peak-inverse voltage seen across any diode is 390.32V.

16. In the given circuit, for maximum power to be delivered to R_L, its value should be __________ Ω. (Round off to 2 decimal places).



Key: **(1.414)**

Sol: In the following network we need to obtain valved R_L for maximum power transfer

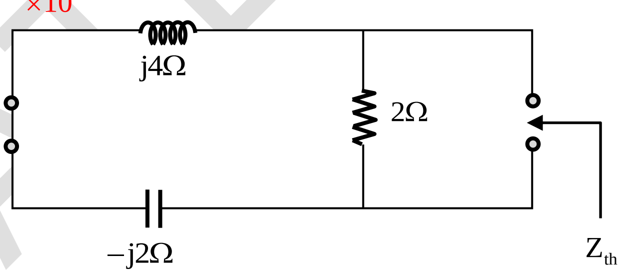


Since the load is purely resistance the condition for maximum power transfer is $R_L = |Z_{th}|$

Computation of Z_{th} independent voltage source is short circuited.

$$Z_L = j\omega L = j \times 1000 \times 4 \times 10^{-3} = j4\Omega$$

$$Z_C = \frac{-1}{\omega C} = \frac{-j}{0.5 \times 10^{-3} \times 10^{3}} = -j2\Omega$$



$$Z_{th} = 2 \,||\, (j4 - j2) = 2 \,||\, j2 = \frac{j4}{2 + j2} = \frac{j2}{1 + j} = \frac{2\angle 90}{\sqrt{2}\angle 45} = \sqrt{2}\angle 45°\Omega$$

$$R_L = |Z_{th}| = \left|\sqrt{2}\angle 45\right| = \sqrt{2} = 1.414\Omega$$

17. Suppose the probability that a coin toss shown "head" is p, where 0 < p < 1. The coin is tossed repeatedly until the first "head" appears. The expected number of tosses required is

(A) $\frac{1}{p^2}$ (B) $\frac{1-p}{p}$ (C) $\frac{1}{p}$ (D) $\frac{p}{(1-p)}$

Key: **(C)**

Sol: Let $p = P_r(\text{head})$

$q = P_r(\text{tail}) = 1 - p$

Let X be a random variable denote number of tosses till to get one head then X = 1, 2, 3, … is a discrete variable and

$X = x$	$P(x)$
1	p
2	qp
3	qqp
⋮	⋮

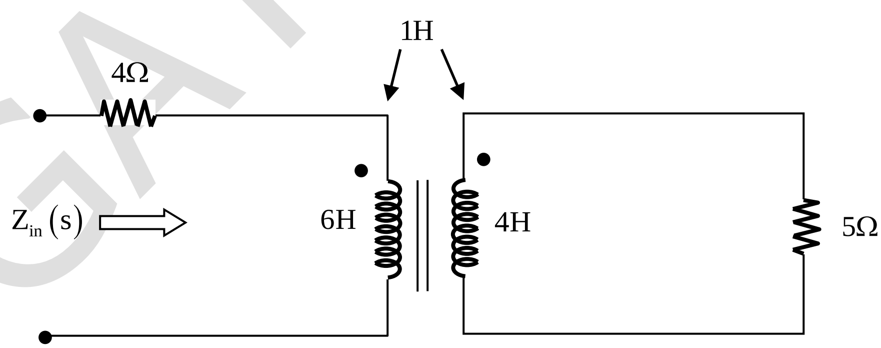
$$\therefore E(x) = \sum_{x=1}^{\infty} x.P(x)$$

$$= 1\times p + 2\times qp + 3\times q^2 p + \ldots$$

$$= p\times\left[1 + 2q + 3q^2 + \ldots\right] = p\times(1-q)^{-2}$$

$$= p\times p^{-2} = \frac{1}{p}$$

18. The input impedance, $Z_{in}(s)$, for the network shown is

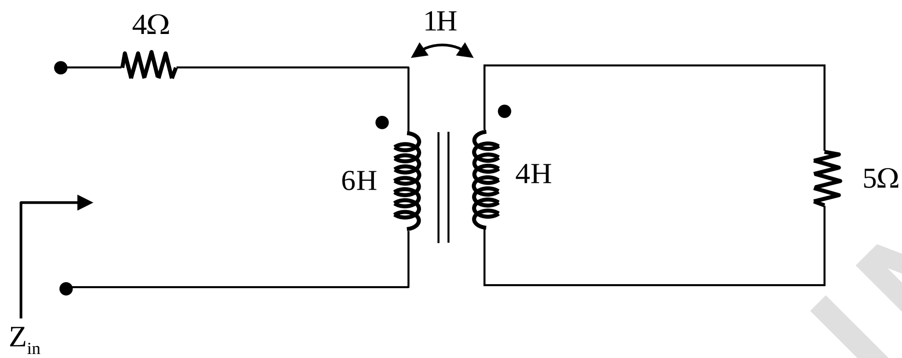


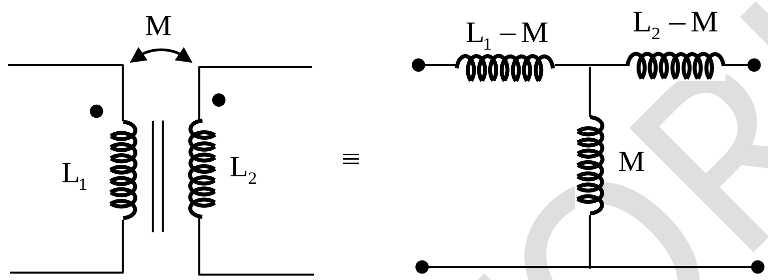
(A) $6s+4$ (B) $7s+4$

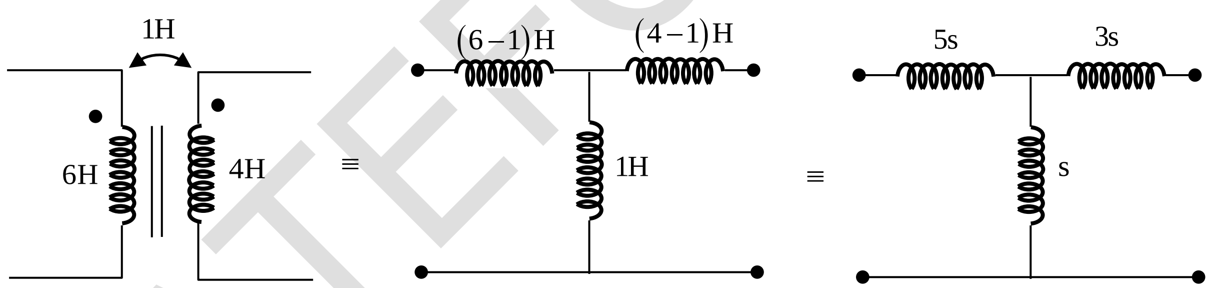
(C) $\frac{25s^2+46s+20}{4s+5}$ (D) $\frac{23s^2+46s+20}{4s+5}$

Key: **(D)**

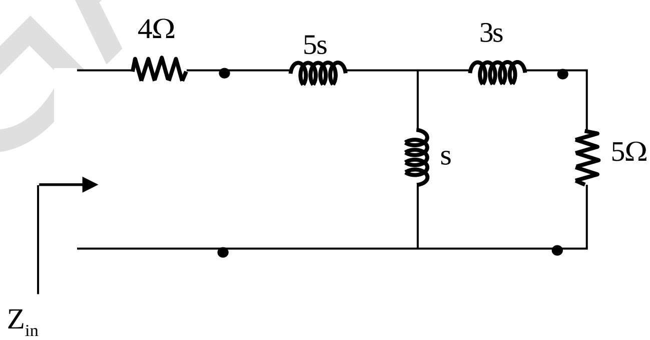
Sol: In the following circuit we need to obtain input impedance.





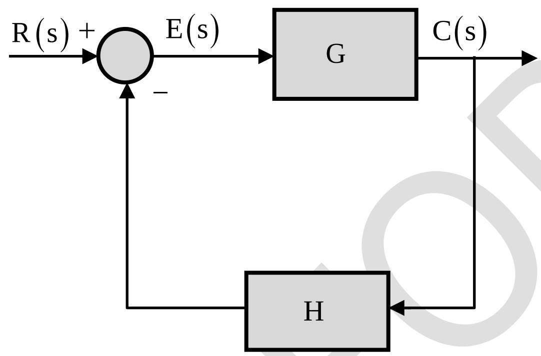


The original network in S domain becomes



$$Z_{in} = (4+5s) + s \parallel (3s+5)$$
$$= 4+5s+\frac{3s^2+5s}{4s+5}$$
$$= \frac{16s+20+20s^2+25s+3s^2+5s}{4s+5}$$
$$= \frac{23s^2+46s+20}{4s+5}$$
$$Z_{in} = \frac{23s^2+46s+20}{4s+5}$$

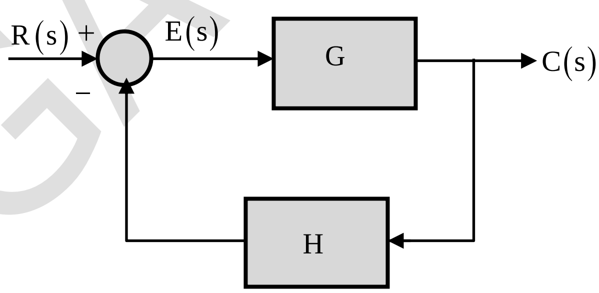
19. For the closed-loop system shown, the transfer function $\frac{E(s)}{R(s)}$ is



(A) $\frac{GH}{1+GH}$ (B) $\frac{G}{1+GH}$ (C) $\frac{1}{1+G}$ (D) $\frac{1}{1+GH}$

Key: (D)

Sol: We need to obtain $\frac{E(s)}{R(s)}$ for following block diagram

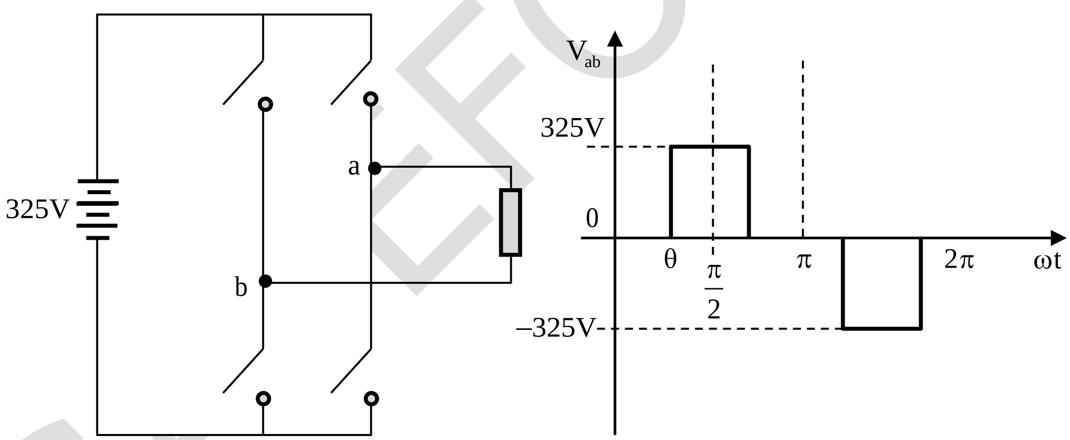


$$\frac{C(s)}{R(s)} = \frac{G}{1+GH}$$ (standard negative feedback system)

$$E(s) = R(s) - C(s)H$$
$$= R(s) - \frac{GH}{1+GH}R(s)$$
$$= R(s)\left[1 - \frac{GH}{1+GH}\right]$$
$$= R(s)\left[\frac{1+GH-GH}{1+GH}\right]$$
$$= R(s)\frac{1}{1+GH}$$
$$\Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1+GH}$$

20. A single-phase full-bridge inverter fed by a 325 V DC produces a symmetric quasi-square waveform across 'ab' as shown. To achieve a modulation index of 0.8, the angle θ expressed in degrees should be ________ (Round off to 2 decimal places).

(Modulation index is defined as the ratio of the peak of the fundamental component of V_{ab} to the applied DC value).



Key: **(51.07)**

Sol: Given,

$$V_{dc} = 325V, \qquad m(\text{Need to achieve}) = 0.8$$

$$V_{ab} = \sum_{1,3,5}^{\infty} \frac{4V_{dc}}{n\pi}\cos n\theta \sin n(\omega t)$$

$$(V_{ab})_{peak\,01} = \frac{4 \times V_{dc}}{\pi}\cos\theta$$

$$m = \frac{(V_{ab01})_{peak}}{V_{ac}} = \frac{\frac{4}{\pi}V_{dc}\cos\theta}{V_{dc}} \Rightarrow 0.8 = \frac{4}{\pi}\cos\theta$$

$\theta = 51.07°$

$\therefore$ To achieve a modulation index of 0.8, the angle θ expressed in degrees should be $51.07°$.

21. Let $f(x)$ be a real-valued function such that $f'(x_0)=0$ for some $x_0 \in (0,1)$, and $f''(x)>0$ for all $x \in (0,1)$. The $f(x)$ has

(A) no local minimum in (0, 1)
(B) one local maximum in (0, 1)
(C) exactly one local minimum in (0, 1)
(D) two distinct local minima in (0, 1)

Key: **(C)**

Sol: Since $f'(x_0)=0 \Rightarrow x_0$ is called stationary point and $x_0 \in (0,1)$

$f''(x) > 0$ for all $x \in (0,1)$

$\Rightarrow f''(x_o) > 0$

$\Rightarrow$ f(x) has local minimum at x_o

i.e., exactly one local minimum in (0, 1)

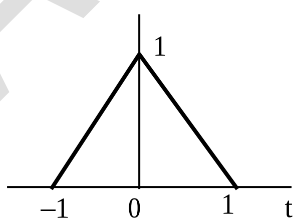
22. Consider a continuous-time signal $x(t)$ defined by $x(t)=0$ for $|t|>1$, and $x(t)=1-|t|$ for $|t|\le 1$. Let the Fourier transform of x(t) defined as $X(\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt.$. The maximum magnitude of $X(\omega)$ is _________.

Key: **(1)**

Sol: Signal: CTFT

$x(t) = 1-|t|; |t| \le 1$

$x(t)$:



$$X(\omega) = (\text{Area of triangle})Sa^2\left(\omega\frac{\text{width}}{4}\right) = \left(\frac{1}{2}\times 2\times 1\right)Sa^2\left(\frac{\omega}{2}\right) = Sa^2\left(\frac{\omega}{2}\right)$$

$\Rightarrow X(0) = Sa^2(0)$

$\Rightarrow X(0) = 1$

23. One coulomb of point charge moving with a uniform velocity $10\,\hat{X}$ m/s enters the region $x \geq 0$ having a magnetic flux density $\vec{B} = (10y\hat{x} + 10x\hat{y} + 10\hat{z})$T. The magnitude for force on the charge at $x = 0^+$ is _______ N. ($\hat{x}$, $\hat{y}$ and $\hat{z}$ are unit vectors along x-axis and z-axis, respectively).

Key: (100)

Sol: The Lorentz magnetic force F due to change moving with velocity $\vec{V}$ in a magnetic field $\vec{B}$ is given by

$\vec{F} = q(\vec{V} * \vec{B})$

Given, $q = 1C, \vec{V} = 10\hat{x}\ m/sec, \vec{B} = (10y\hat{x} + 10x\hat{y} + 10\hat{z})$

$\Rightarrow \vec{F} = 1\left(10\hat{x} \times \left(10y\hat{x} + 10x\hat{y} + 10\hat{z}\right)\right) = 100x(\hat{x} \times \hat{y}) + 100(\hat{x} \times \hat{x})$

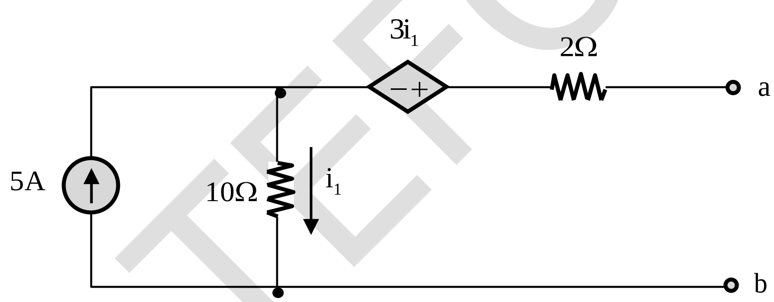
$\vec{F} = 100x\hat{z} + 100(\hat{y})$

At $x = 0^+$

$\vec{F} = -100\hat{y}$

$\Rightarrow$ Magnitude of $\vec{F} = 100N$

24. For the network shown, the equivalent Thevenin voltage and Thevenin impedance as seen terminals 'ab' is

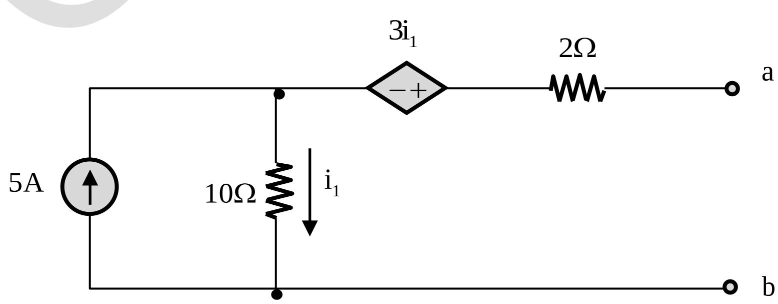


(A) 65 V in series with $15\,\Omega$ (B) 35V in series with $2\,\Omega$

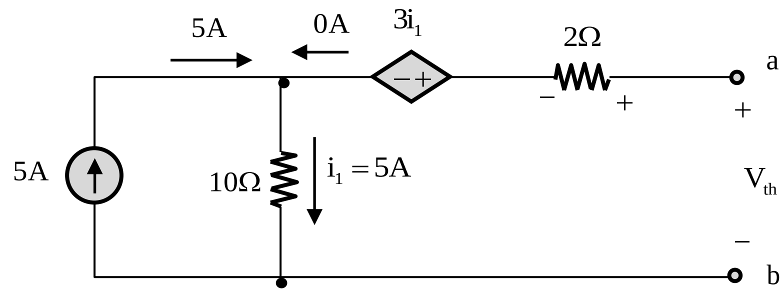
(C) 10 V in series with $12\,\Omega$ (D) 50V in series with $2\,\Omega$

Key: (A)

Sol: In the following network we need to obtain V_{th} and R_{th}

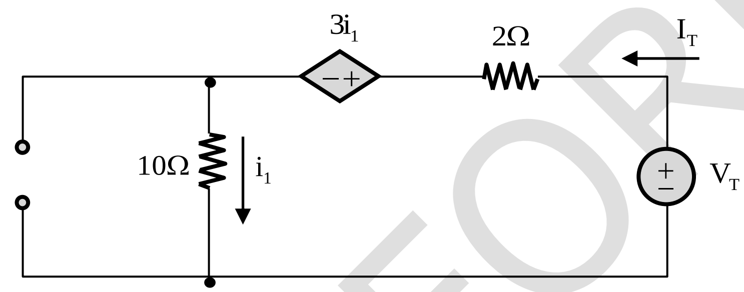


Calculation of V_{th}



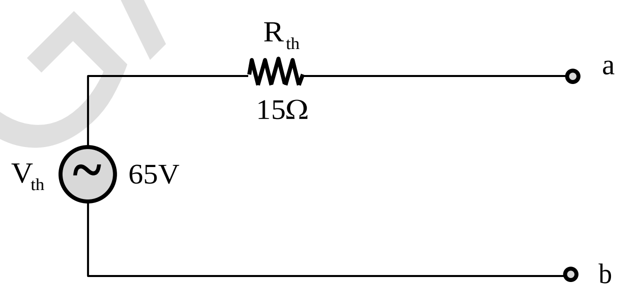
$$V_{th} - 0 - 3i_1 - 10i_1 = 0$$
$$V_{th} = 13i_1 = 13 \times 5 = 65V$$

Calculation of R_{th}



(Current source is opened, dependent source is not disturbed)

$$V_T - 2I_T - 3i_1 - 10i_1 = 0$$
$$V_T = 2I_T + 13i_1$$
$$V_T = 15I_T \qquad (\because i_1 = I_T)$$
$$\frac{V_T}{I_T} = 15\Omega$$
$$R_{th} = 15\Omega$$



$[V_{th} = 65V, R_{th} = 15\Omega]$

25. Consider a power system consisting of N number of buses. Buses in this power system are categorized into slack bus, PV buses and PQ buses for load flow study. The number of PQ buses is N_L. The balanced Newton-Raphson method is used to carry out load flow study in polar form. H, S, M and R are sub-matrices of the Jacobian matrix J as shown below.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} \text{ where } J = \begin{bmatrix} H & S \\ M & R \end{bmatrix}$$

The dimension of the sub-matrix M is

(A) $N_L \times (N-1+N_L)$

(B) $(N-1) \times (N-1+N_L)$

(C) $N_L \times (N-1)$

(D) $(N-1) \times (N-1-N_L)$

Key: (C)

Sol: We know, Jacobian matrix is given by

$$J = \begin{bmatrix} H = \frac{\partial P}{\partial \delta} & S = \frac{\partial P}{\partial |V|} \\ M = \frac{\partial Q}{\partial \delta} & R = \frac{\partial Q}{\partial |V|} \end{bmatrix}$$

Where numerator part of each sub matrices talks about known quantities and denominator about unknown.

Here for $M = \frac{\partial Q}{\partial \delta}$

Q is known for PQ buses i.e., N_L (given) and S is unknown for all buses except slack buses i.e., (N-1)

Hence, dimension of M should be $N_L \times (N-1)$

26. The power input to a 500V, 50 Hz, 6-pole, 3-phae induction motor running at 975 RPM is 40 kW. The total stator losses are 1 kW. If the total friction and windage losses are 2.25 kW, then the efficiency is ________%.

Key: **(90)**

Sol: Given,

$$\left.\begin{aligned} P &= 6 \\ V &= 500V, \\ f &= 50Hz \\ P_i &= 40kW \\ N_r &= 975 \text{ rpm} \\ f \text{ and } W \text{ loss} &= 2.025 \text{ kW} \end{aligned}\right\} \eta_m = ?$$

Synchronous speed,

$$N_S = \frac{120f}{P} = \frac{120\times 50}{60}$$
$$N_S = 1000 \text{ rpm}$$

$$S = \frac{N_S - N_r}{N_S} = \frac{1000-975}{1000} = 0.025$$

$$P_{gm} = (1-S)P_i = (1-0.025)\times(P_{in} - \text{stator loss})$$
$$= 0.975\times(40-1)$$

$$P_{gm} = 38.025 \text{ kW}$$

Motor efficiency,

$$\eta_m = \frac{P_{out}}{P_{in}} = \frac{P_{gm} - P_{mech}}{P_{in}} = \frac{38.025-2.025}{40}$$
$$\eta_m = 0.9 = 90\%$$

27. A 1 μC point charge is held at the origin of a Cartesian coordinate system. If a second point charge of 10 μC is moved from (0, 10, 0) to (5, 5, 5) and subsequently to (5, 0, 0), then the total work done is ________ mJ. (Round off to 2 decimal places).

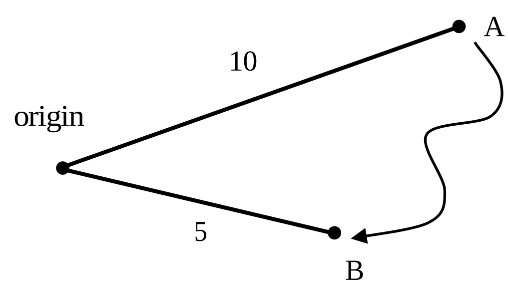
Take $\frac{1}{4\pi\varepsilon_o} = 9\times 10^9$ in SI units. All coordinates are in meters.

Key: (9)

Sol: Work done depends on initial and final position

Initial point $\rightarrow (0,10,0)$ distance from origin = 10

Final point $\rightarrow (5,0,0)$ distance from origin = 5



$$W_{AB} = -q_2(V_A - V_B)$$

$$V_A = \frac{q_1}{4\pi\varepsilon_o r_A} = \frac{1\times 10^{-6}\times 9\times 10^9}{10} = 9\times 10^2$$

$$V_B = \frac{q_1}{4\pi\varepsilon_o r_B} = \frac{1\times 10^{-6}\times 9\times 10^9}{5} = 18\times 10^2$$

Now $= 10\times 10^{-6}\left[9\times 10^2\right] = 90\times 10^{-4} = 9\times 10^{-3}$ J or 9 mJ

28. A counter is constructed with three D flip-flops. The input-output pairs are named $(D_0, Q_0), (D_1, Q_1)$ and (D_2, Q_2) where the subscript 0 denotes the least significant bit. The output sequence is desired to be the Gray-code sequence 000, 001, 011, 010, 110, 111, 101 and 100 repeating periodically. Note that the bits are listed in the $Q_2Q_1Q_0$ format. The combinational logic expression for D_1 is

(A) $Q_2Q_0 + Q_1\bar{Q}_0$ (B) $Q_2Q_1Q_0$

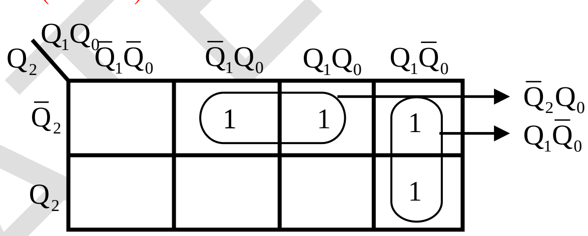
(C) $Q_2Q_1 + \bar{Q}_2\bar{Q}_1$ (D) $\bar{Q}_2Q_0 + Q_1\bar{Q}_0$

Key: **(D)**

Sol: The question is about counter design based on given information we can have the following table.

Present state			Next state			Flip flop inputs		
Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	D_2	D_1	D_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	1	0	1	1
0	1	0	1	1	0	1	1	0
0	1	1	0	1	0	0	1	0
1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	1	0	0
1	1	0	1	1	1	1	1	1
1	1	1	1	0	1	1	0	1

$D_1(Q_2, Q_1, Q_0) = \Sigma m(1, 2, 3, 6)$



$D_1 = \bar{Q}_2Q_0 + Q_1\bar{Q}_0$

29. The causal signal with z-transform $z^2(z-a)^{-2}$ is (u[n] is the unit step signal)

(A) $a^{2n}u[n]$ (B) $n^{-1}a^nu[n]$ (C) $n^2a^nu[n]$ (D) $(n+1)a^nu[n]$

Key: **(D)**

Sol: Given $X(z) = \frac{z^2}{(z-a)^2} = \frac{1}{(1-az^{-1})^2}$

We know, $\frac{1}{(1-az^{-1})^2} \leftrightarrow (n+1)a^nu(n)$ (standard z-transform)

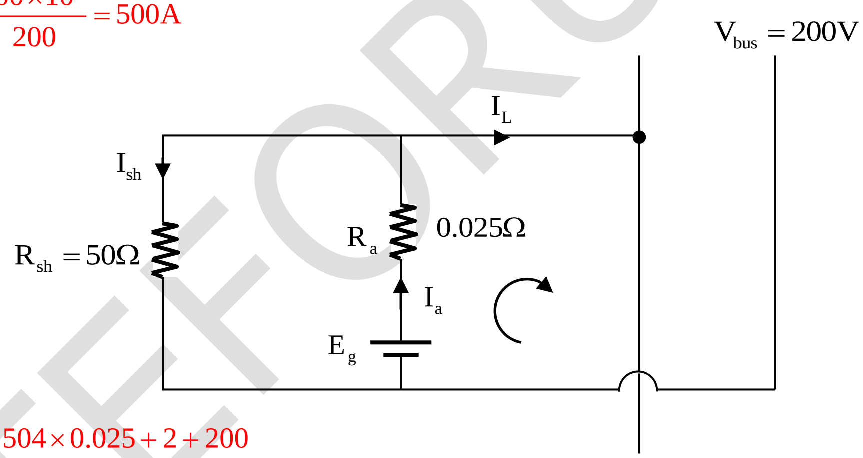
30. A belt-driven DC shunt generator running at 300 RPM delivers 100 kW to 200V DC grid. It continues to run as a motor when the belt breaks, taking 10 kW from the DC grid. The armature resistance is $0.025\,\Omega$, field resistance is $50\,\Omega$, and brush drop is 2V. Ignoring armature reaction, the speed of the motor is ____________RPM. (Round off to 2 decimal places).

Key: **(275.18)**

Sol: Given,

$$\left.\begin{aligned} N &= 300 \text{ rpm} \\ P_0 &= 100 \text{ kW} \\ V &= 200V \\ R_a &= 0.025\Omega \\ R_{sh} &= 50\Omega \\ \text{Brush drop} &= 2V \end{aligned}\right\} N = ?$$

Line current, $I_L = \frac{P_0}{V} = \frac{100\times10^3}{200} = 500A$



$I_{sh} = \frac{V}{R_{sh}} = \frac{200}{50} = 4A$

$I_a = I_L + I_{sh}$

$I_a = 500 + 4$

$I_a = 504A$

Apply KVL

$E_g - I_aR_a - B.D = V_{Bus}$

$E_g = I_aR_a + B.D + V_{Bus} = 504\times0.025 + 2 + 200$

$E_g = 214.6V$

If belt brokes,

$P = 10\times10^3$

$I_L = \frac{P}{V} = 50A$

$I_{sh} = \frac{V}{R_{sh}} = \frac{200}{50} = 4A$

$I_a = I_L - I_{sh} = 50 - 4$

$I_a = 46A$

By KVL,

$V - I_a R_a - BD = E_b$

$200 - 46 \times 0.025 = E_b$

$E_b = 196.85V$

$N \alpha \frac{E_b}{\phi}$

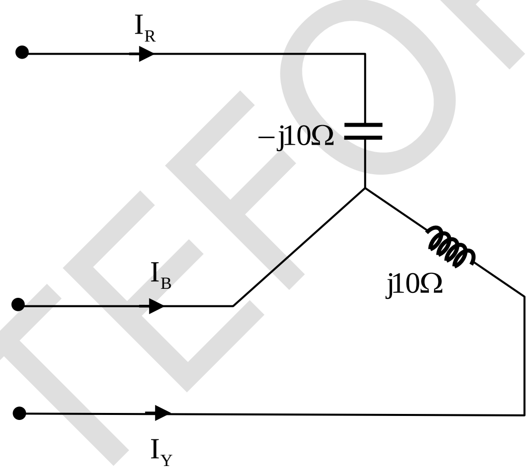
'ϕ' is constant since current through shunt field winding is constant

$\therefore \frac{N_m}{N_g} = \frac{E_m}{E_g}$

$N_m = \frac{E_m}{E_g} \times N_g = \frac{196.85}{214.6} \times 300$

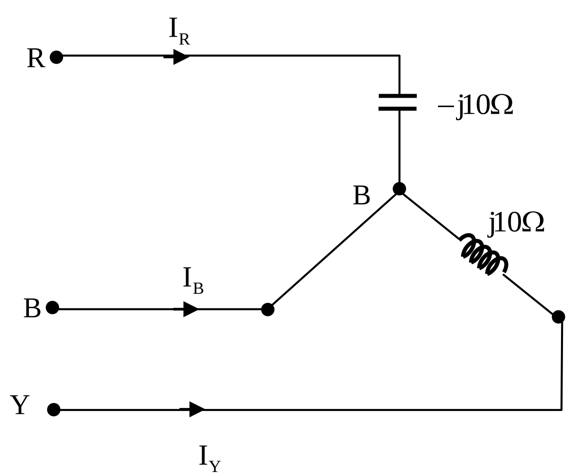
$N_m = 275.18\,\text{rpm}$

31. A three-phase balanced voltage is applied to the load shown. The phase sequence is RYB. The ratio $\frac{|I_B|}{|I_R|}$ is ____________.



Key: (1)

Sol: In the following network we need to obtain $\left|\frac{I_B}{I_R}\right|$



Since the source is balanced

$V_{RY} = V\angle 0$

$V_{YB} = V\angle -120$

$V_{BR} = V\angle 120$

$I_R = \frac{V_{RB}}{-j10} = \frac{-V_{BR}}{-j10} = \frac{-V\angle 120}{-j10} = \frac{V}{10}\angle 30$

$I_Y = \frac{V_{YB}}{j10} = \frac{V\angle -120}{j\angle 10} = \frac{V}{10}\angle -210°$

$I_R + I_Y + I_B = 0$

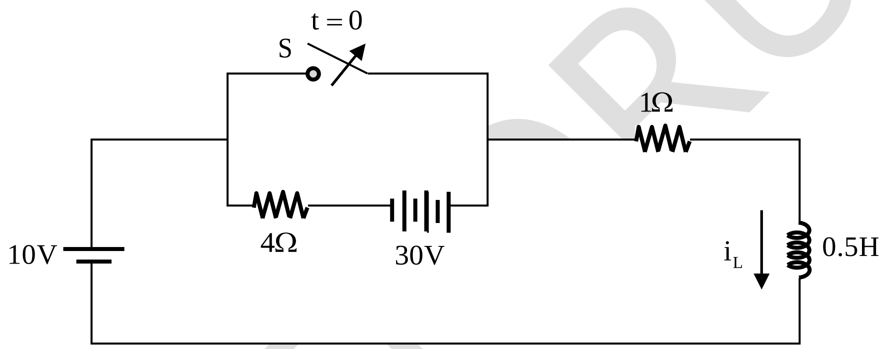
$$\Rightarrow I_B = -\left[I_R + I_y\right] = \left[\frac{V}{10}\angle 30 + \frac{V}{10}\angle -210^\circ\right] = \frac{-V}{10}\left[1\angle 30^\circ + 1\angle -210\right] = \frac{-V}{10}\angle 90^\circ$$

$$|I_R| = \left|\frac{V}{10}\angle 30\right| = \frac{V}{10}$$

$$|I_B| = \left|\frac{-V}{10}\angle 90\right| = \frac{V}{10}$$

$$\left|\frac{I_B}{I_R}\right| = 1$$

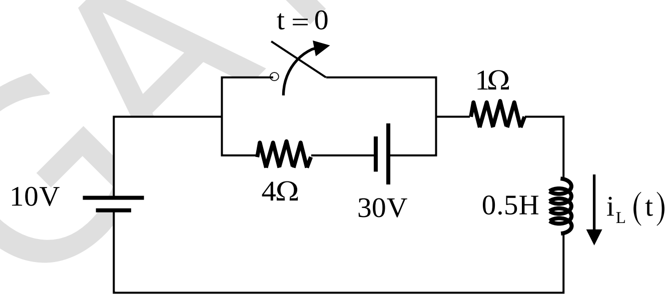
32. In the circuit, switch 'S' is in the closed position for a very long time. If the switch is opened at time t = 0, then $i_L(t)$ in amperes, for $t \geq 0$ is



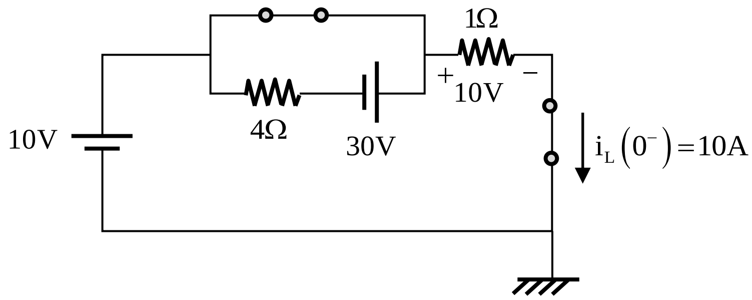
(A) $8e^{-10}$ (B) 10 (C) $10\left(1-e^{-2t}\right)$ (D) $8+2e^{-10t}$

Key: **(D)**

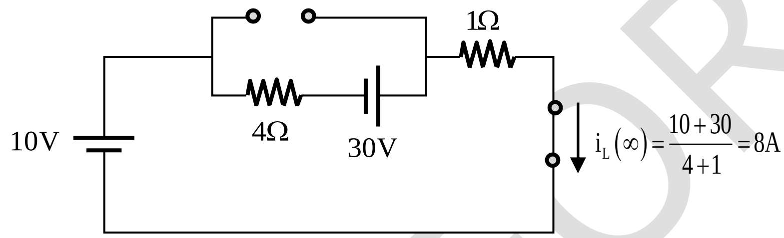
Sol: In the following network we need to obtain $i_L(t)$ for $t \geq 0$



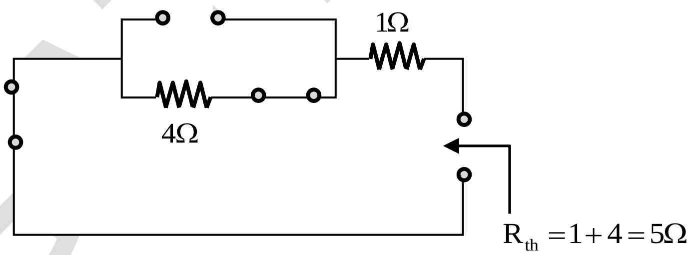
at $t = 0^-$ → Network in steady state → Switch is closed → Inductor is short circuited



at $t = \infty$
- Network is in steady state
- Switch is open
- Inductor is short circuited



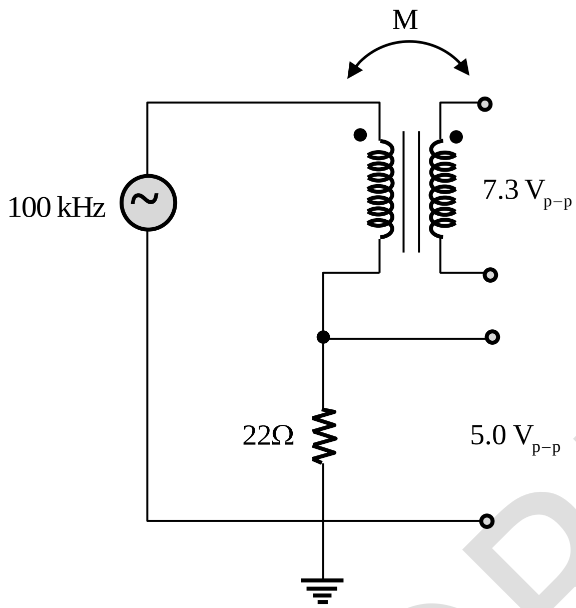
R_{th}
- Network for $t > 0$ is to be referred
- Independent voltage source to be shorted



Time constant $= \frac{L}{R_{th}} = \frac{0.5}{5} = \frac{1}{10}$

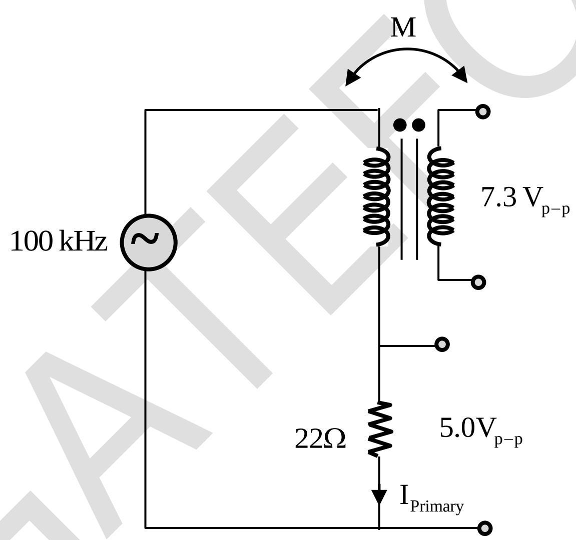
$$i_L(t) = i_L(\infty) + \left[i_L(0^-) - i_L(\infty)\right]e^{-t/\tau};\ t \geq 0 = 8 + [10-8]e^{-10t} = 8 + 2e^{-10t};\ t \geq 0$$

33. An air core radio-frequency transformer as shown has a primary winding and a secondary winding. The mutual inductance M between the windings of the transformer is ___________ μH. (Round off to 2 decimal places).



Key: (51.1)

Sol: Given,



We need to calculate mutual inductance M between the windings of the transformer.

$\because$ Secondary is open circuited,

$I_{sec} = 0A$

$\therefore (7.3)_{p-p} = |\omega M I_{primary}|$

$\Rightarrow 7.3 = 2\times\pi\times100\times10^3\times M\times\frac{5}{22} = 2\times\frac{22}{7}\times100\times10^3\times M\times\frac{5}{22} \Rightarrow \boxed{M = 51.1\,\mu H}$

Hence, the mutual inductance M between the windings of the transformer is $51.1\,\mu H$.

34. Let p and q be real numbers such that $p^2+q^2=1$. The eigen values of the matrix $\begin{bmatrix} p & q \\ q & -p \end{bmatrix}$ are

(A) j and –j (B) 1 and 1 (C) 1 and –1 (D) pq and –pq

Key: (C)

Sol: Characteristic equation of the matrix is

$$\lambda^2-\lambda\times(p-p)+\left(-p^2-q^2\right)=0$$

$$\Rightarrow \lambda^2-1=0\Rightarrow\lambda=-1,1\ \left(\text{since } p^2+q^2=1\right)$$

$$\left(\text{Hint}:\lambda^2-\lambda\times(\text{trace})+(\text{determinant of matrix})\right)$$

35. Let $p(z)=z^3+(1+j)z^2+(2+j)z+3$, where Z is a complex number. Which one of the following is true?

(A) All the roots cannot be real

(B) conjugate {p(z)} = p(conjugate {z}) for all z

(C) The sum of the roots of p(z) = 0 is a real number

(D) The complex roots of the equation p(z) = 0 come in conjugate pairs

Key: (D)

Sol: Let z = j be a complex number

$$\Rightarrow \bar{z}=-j$$

$$p(z)=j^3+(1+j)j^2+(2+j)j+3=-j-1-j+2j-1+3=1$$

$$\Rightarrow \overline{p(z)}=1$$

And $p(\bar{z})=p(-j)=(-j)^3+(1+j)(-j)^2+(2+j)(-j)+3$

$$=j-1-j-2j+1+3=3-2j$$

$$\Rightarrow \overline{p(z)}\neq p(\bar{z})$$

$\therefore$ option 'B' is not true.

(A) We can't say all the roots are real

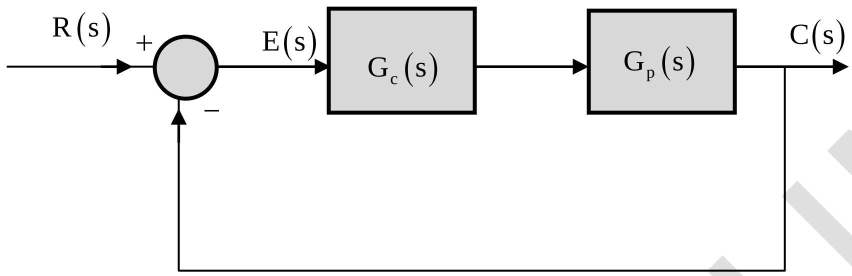
Eg: Let $p(z)=z^3-jz^2+z-j$ has a 3 complex roots $z=-j, j, j$

So, option 'A' and 'C' are not true.

We know that complex roots occur in conjugate pairs.

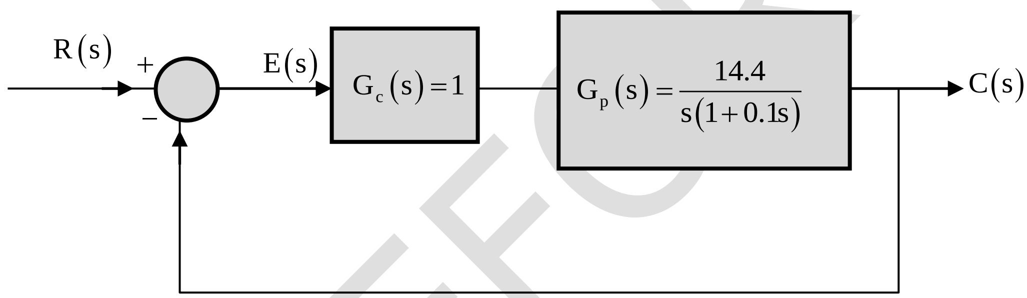
$\therefore$ option 'D' is answer.

36. Consider a closed-loop system as shown. $G_p(s)=\dfrac{14.4}{s(1+0.1s)}$ is the plant transfer function and $G_c(s)=1$ is the compensator. For a unit-step input, the output response has damped oscillations. The damped natural frequency is ____________rad/s. (Round off to 2 decimal places).



Key: **(10.90)**

Sol: The given block diagram is as follows



It is given that for unit step input response has damped oscillation we need to obtain damped natural frequency.

The closed loop transfer function of above block diagram is

$$\frac{C(s)}{R(s)}=\frac{G(s)G(s)}{1+G_c(s)G_p(s)}=\frac{\dfrac{14.4}{s(1+0.1s)}}{1+\dfrac{14.4}{s(1+0.1s)}}=\frac{14.4}{s(1+0.1s)+14.4}$$

$$=\frac{14.4}{s+0.1s^2+14.4}=\frac{144}{s^2+10s+144}=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_0^2}$$

By comparison $\omega_n^2=144\Rightarrow\omega_n=12\text{ rad/sec}$

$$2\xi\omega_n=10\Rightarrow\xi=\frac{10}{2\omega_n}=\frac{10}{24}=0.416$$

Damped natural frequency: $\omega_d=\omega_n\sqrt{1-\xi^2}=12\sqrt{1-(0.416)^2}=10.90\text{ rad/sec}$

37. Two single-cover power cables have total conductor resistances of 0.7 Ω and $0.5\,\Omega$, respectively, and their insulation resistance (between core and sheath) are 600 MΩ and 900 MΩ respectively. When the two cables are joined in series, the ratio of insulation resistance to conductor resistance is ________$\times 10^6$.

Key: **(300)**

Sol: $R_{C(total)} = R_{C1} + R_{C2}(\text{series}) = 0.7 + 0.5 = 1.2$

$$R_{ins\ total} = \frac{600 \times 900}{600 + 900} = 360 \times 10^6\,\Omega$$

$$\frac{R_{ins(total)}}{R_{C(total)}} = \frac{360 \times 10^6}{1.2} = 300 \times 10^6$$

38. An 8-pole, 50 Hz, three-phase, slip-ring induction motor has an effective rotor resistance of 0.08 Ω per phase. Its speed at maximum torque is 650 RPM. The additional resistance per phase that must be inserted in the rotor to achieve maximum torque at start is ____________Ω. (Round off to 2 decimal places). Neglect magnetizing current and stator leakage impedance. Consider equivalent circuit parameters referred to stator.

Key: **(0.52)**

Sol: Given,

$$\left.\begin{aligned} &P = 8, \\ &f = 50\text{Hz}, \\ &R_2 = 0.08\,\Omega/\text{ph} \\ &N_{Tm} = 650\text{ rpm} \end{aligned}\right\} R_{add} = ?$$

Synchronous speed, $N_S = \frac{120f}{P} = \frac{120 \times 50}{8} = 750\text{ rpm}$

Slip at maximum torque,

$$S_{Tm} = \frac{N_S - N_{Tm}}{N_S} = \frac{750 - 650}{750} = 0.133$$

We also have,

$$S_{Tm} = \frac{R'_2}{X'_2} \Rightarrow X'_2 = \frac{R'_2}{ST_m} = \frac{0.08}{0.133} = 0.6\Omega$$

For producing maximum torque at starting

$S_{Tm} = 1$

$$\Rightarrow \frac{R'_{2(new)}}{X'_2} = 1 \Rightarrow R'_{2(new)} = X'_2 = 0.6$$

$R_{ext} = 0.6 - R'_2 = 0.6 - 0.08$

$R_{ext} = 0.52\Omega$

39. Consider a large parallel plate capacitor., The gap d between the two plates is filled entirely with dielectric slab of relative permittivity 5. The plates are initially charged to a potential different of V volts and then disconnected from the source. If the dielectric slab is pulled out completely, then the ratio of the new electric field E_2 in the gap to the original electric field E_1 is _________.

Key: **(5)**

Sol: On removal of voltage source, charge on the capacitor will remain constant.

Initially, $Q_1 = \frac{\varepsilon_o 5A}{d}.v_1 \quad (Q = CV)\ldots(1)$

After removing dielectric slab and voltage source

$$Q_1 = \frac{\varepsilon_o.A}{d}.V_2 \qquad \ldots(2)$$

V_1 and V_2 are potential between plates

Electric field $E = \frac{V}{d} \Rightarrow \frac{E_1}{E_2} = \frac{V_1}{V_2}$

$E_1 \rightarrow$ original electric field

$E_2 \rightarrow$ new electric field

Since (1) = (2), $\frac{\varepsilon_o 5.A}{d}V_1 = \frac{\varepsilon_o.A}{d}V_2$

$$\frac{V_1}{V_2} = \frac{1}{5}$$

$\Rightarrow \frac{E_1}{E_2} = \frac{1}{5}$ or ratio of new electric field strength to original electric field strength = 5

40. Two generators have cost functions F_1 and F_2. Their incremental-cost characteristics are

$$\frac{dF_1}{dP_1} = 40 + 0.2P_1$$

$$\frac{dF_2}{dP_2} = 32 + 0.4P_2$$

They need to deliver a combined load of 260 MW. Ignoring the network losses, for economic operation, the generations P_1 and P_2 (in MW) are

(A) $P_1 = P_2 = 130$

(B) $P_1 = 160, P_2 = 100$

(C) $P_1 = 140, P_2 = 120$

(D) $P_1 = 120, P_2 = 140$

Key: **(B)**

Sol: $\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$

$$40 + 0.2P_1 = 32 + 0.4P_2$$

$$40 - 32 = 0.4P_2 - 0.2P_1$$

$8 = 0.4P_2 - 0.2P_1$ ...(1)

$P_1 + P_2 = 260\text{ MW}$ ...(2)

$\Rightarrow P_1 - 2P_2 = -40$ ...(1)

$\Rightarrow P_1 + P_2 = 260$ ...(2)

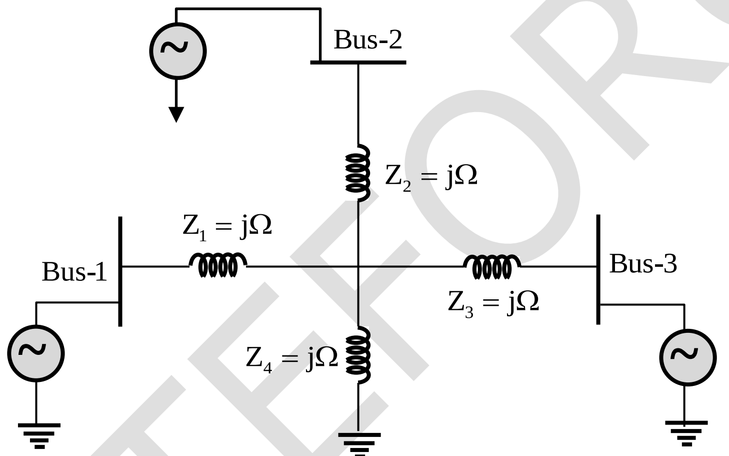
Solve (1) and (2) for finding the values of P_1 and P_2

$-3P_2 = -300$

$P_2 = 100\text{MW}$

$P_1 = 160\text{MW}$

41. A 3-Bus network is shown. Consider generators as ideal voltage sources. If rows 1, 2 and 3 of the Y_{Bus} matrix correspond to Bus 1, 2 and 3, respectively then Y_{Bus} of the network is



(A) $\begin{bmatrix} -4j & 2j & 2j \\ 2j & -4j & 2j \\ 2j & 2j & -4j \end{bmatrix}$

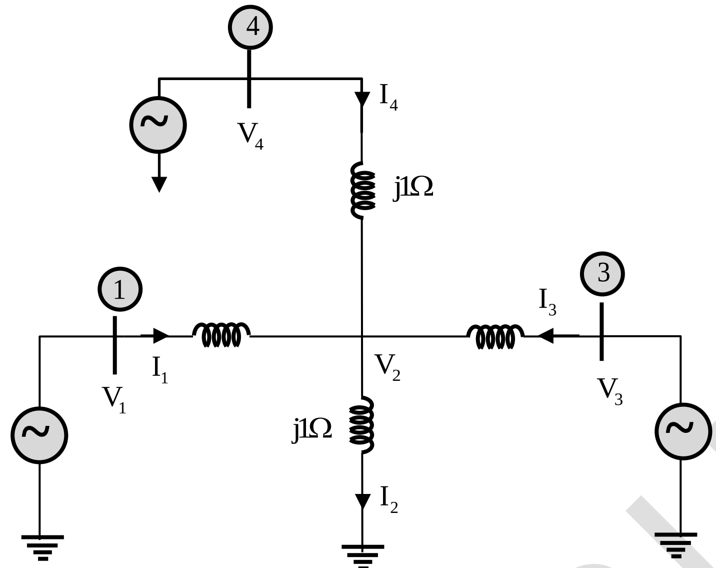
(B) $\begin{bmatrix} -\frac{1}{2}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{1}{2}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & -\frac{1}{2}j \end{bmatrix}$

(C) $\begin{bmatrix} -4j & j & j \\ j & -4j & j \\ j & j & -4j \end{bmatrix}$

(D) $\begin{bmatrix} -\frac{3}{4}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{3}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & -\frac{3}{4}j \end{bmatrix}$

Key: (D)

Sol:



All voltage sources are ideal

$$I_1 + I_3 + I_4 = I_2 \quad \ldots(1)$$

$$I_1 = (V_1 - V_2)(-j1)$$

$$I_1 = (-j1)V_1 + (j1)V_2 \quad \ldots(2)$$

$$I_2 = V_2(-j1) \quad \ldots(3)$$

$$I_3 = (-j1)V_2 + (j1)V_3 \quad \ldots(4)$$

$$I_4 = (j1)V_2 - (j1)V_4 \quad \ldots(5)$$

$$I_4 = (V_4 - V_2)(-j1)$$

$$I_4 = (j1)V_2 - (j1)V_4 \quad -(5)$$

From equation (1) and (3)

$$I_1 + I_3 + I_4 = V_2(-j1)$$

$$V_2 = (j1)\left[(-j1)V_1 + (j1)V_2 + (j1)V_2 + (-j1)V_3 + (j1)V_2 - (j1)V_4\right]$$

$$V_2 = V_1 - 3V_2 + V_3 + V_4$$

$$4V_2 = V_1 + V_3 + V_4$$

$$V_2 = \frac{1}{4}V_1 + \frac{1}{4}V_3 + \frac{1}{4}V_4 \quad \ldots(6)$$

$$I_1 = (j1)V_1 + (j1)\left[\frac{1}{4}V_1 + \frac{1}{4}V_3 + \frac{1}{4}V_4\right]$$

$$I_1 = -j\frac{3}{4}V_1 + j\frac{1}{3}V_3 + j\frac{1}{4}V_4 - (7)$$

$$I_3 = (-j1)\left[V_3 - \frac{1}{4}V_1 - \frac{1}{4}V_3 - \frac{1}{4}V_4\right]$$

$$I_3 = j\frac{1}{4}V_1 - j\frac{3}{4}V_3 + j\frac{1}{4}V_4 \qquad \ldots(8)$$

$$I_4 = (j1)\left[\frac{1}{4}V_1 + \frac{1}{4}V_3 + \frac{1}{4}V_4\right] - (j1)V_4$$

$$I_4 = j\frac{1}{4}V_1 + j\frac{1}{4}V_3 - j\frac{3}{4}V_4 \qquad \ldots(9)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{3}{4}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{3}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & -\frac{3}{4}j \end{bmatrix}}_{Y_{Bus}} \begin{bmatrix} V_1 \\ V_3 \\ V_4 \end{bmatrix}$$

42. Which one of the following vector functions represents a magnetic field $\vec{B}$?

($\hat{x}$, $\hat{y}$ and $\hat{z}$ are unit vectors along x-axis, y-axis and z-axis respectively)

(A) $10x\hat{x} + 20y\hat{y} - 30z\hat{z}$

(B) $10y\hat{x} + 20x\hat{y} - 10z\hat{z}$

(C) $10x\hat{x} - 30z\hat{y} + 20y\hat{z}$

(D) $10z\hat{x} + 20y\hat{y} - 30x\hat{z}$

Key: (A)

Sol: By Maxwell's equation we know that divergence of magnetic field in zero

$\nabla.B = 0$

Option 1: $\nabla.B = 10 + 20 - 30 = 0$

Option 2: $\nabla.B = 0 + 0 - 10 = -10$

Option 3: $\nabla.B = 10 + 0 + 0 = 10$

Option 4: $\nabla.B = 0 + 20 - 0 = 20$

$$\nabla.B = \frac{\partial}{\partial x}B_x + \frac{\partial}{\partial y}B_y + \frac{\partial}{\partial y}B_z$$

Thus option (A) is correct answer.

43. The state space representation of a first-order system is given as

$$\dot{x} = -x + u$$
$$y = x$$

Where, x is the state variable, u is the control input and y is the controlled output. Let u = –Kx be the control law, where K is the controller gain. To place closed-loop pole at –2, the value of K is _______.

Key: **(1)**

Sol: It is given that

$$\dot{x} = -x + u$$
$$y = x$$
$$u = -kx$$

We need to obtain value of such that closed loop pole will be at s = –2.

Closed loop pole is roots of characteristic equation,

Characteristic equation in terms of state space is $|sI - A| = 0$

$$\dot{x} = -x + u = -x - kx \qquad (\because u = -kx)$$

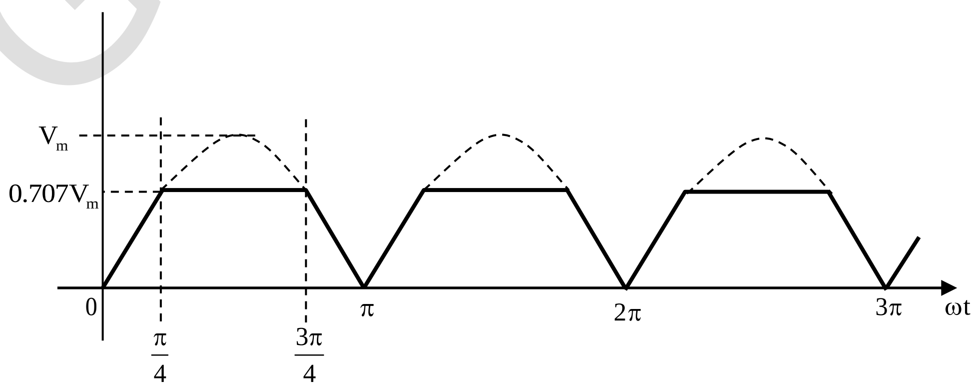
$$\Rightarrow sX(s) = -X(s) - kX(s) \qquad \Rightarrow \dot{x} = x(-k-1)$$
$$\Rightarrow X(s)[s+1-k] = 1 \qquad \Rightarrow \dot{x} = Ax + BU$$

By comparison $A = -k - 1$

Characteristic equation: $|sI - A| = 0$

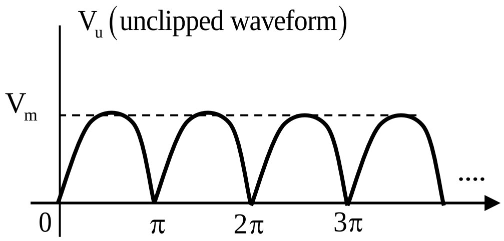
$$|s - (-k-1) = 0|$$
$$|s + k + 1| = 0$$
$$s + k + 1 = 0 \qquad (\text{Determinant of a constant is constant})$$
$$k + 1 = 2 \qquad (\text{closed loop pole is } k+1 \text{ and if should be at } 2)$$
$$k = 1$$

44. The waveform shown in solid line is obtained by clipping a full-wave rectified sinusoid (shown dashed). The ratio of the RMS value of the full-wave rectified waveform to the RMS value of the clipped waveform is _______(Round off to 2 decimal places).



Key: **(1.21)**

Sol: In this case we need to obtain the ratio of r.m.s values of 2 wave forms.



$$[V_{u_{rms}}]^2 = \frac{1}{T_0}\int_0^{T_0} V_u^2 dt$$

$$= \frac{1}{\pi}\int_0^{\pi} (V_m \sin t)^2 dt$$

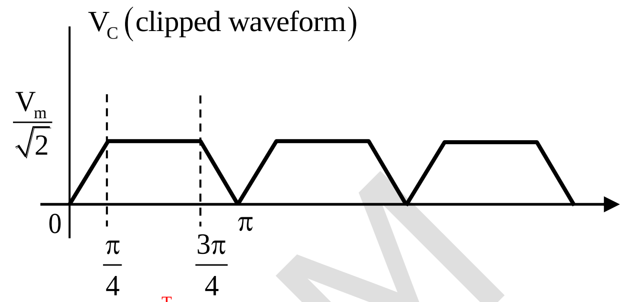
$$= \frac{V_m^2}{2\pi}\int_0^{\pi} (1-\cos 2t) dt$$

$$= \frac{V_m^2}{2\pi}\left[(t)_0^{\pi} - \left(\frac{\sin 2t}{2}\right)_0^{\pi}\right]$$

$$= \frac{V_m^2}{2}\left[(\pi - 0) - \frac{1}{2}(\sin 2\pi - \sin 0)\right] \quad (\sin 2\pi \to 0)$$

$$= \frac{V_m^2}{2}$$

$$V_{u_{rms}} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$



$$(V_{C_{rms}})^2 = \frac{1}{T_o}\int_0^{T_o} V_C^2 dt$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/4} V_m^2 \sin^2 t dt + \int_{\pi/4}^{3\pi/4}\left(\frac{V_m}{\sqrt{2}}\right)^2 dt + \int_{3\pi/4}^{\pi} V_m \sin^2 t dt\right]$$

$$= \frac{V_m^2}{\pi}\left[\int_0^{\pi/4}\frac{1-\cos 2t}{2}dt + \frac{1}{2}\int_{\pi/4}^{3\pi/4} dt + \int_{3\pi/4}^{\pi/4}\left(\frac{1-\cos 2t}{2}dt\right)\right]$$

$$= \frac{V_m^2}{\pi}\left[\int_0^{\pi/4}(1-\cos 2t) dt + \frac{1}{2}\int_{\pi/4}^{3\pi/4} dt\right]$$

$$= \frac{V_m^2}{\pi}\left[(t)_0^{\pi/4} - \left(\frac{\sin 2t}{2}\right)_0^{\pi/4} + \frac{1}{2}(t)_{\pi/4}^{3\pi/4}\right]$$

$$= \frac{V_m^2}{\pi}\left[\frac{\pi}{4} - \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin 0\right) + \frac{1}{2}\left(\frac{3\pi}{4} - \frac{\pi}{4}\right)\right]$$

$$= \frac{V_m^2}{\pi}\left[\frac{\pi}{4} - \frac{1}{2} + \frac{\pi}{4}\right]$$

$$= \frac{V_m^2}{\pi}\left[\frac{\pi}{2} - \frac{1}{2}\right] = \frac{V_m^2}{\pi}\left(\frac{\pi - 1}{2}\right)$$

$$= V_m^2\left(\frac{\pi - 1}{2\pi}\right)$$

$$V_{C_{rms}} = \sqrt{V_m^2\left(\frac{\pi-1}{2\pi}\right)} = 0.5838\, V_m$$

$$\frac{V_{u_{rms}}}{V_{C_{rms}}} = \frac{0.707 V_m}{0.583\, V_m} = 1.21$$

45. Two discrete-time linear time-invariant systems with impulse responses $h_1[n]=\delta[n-1]+\delta[n+1]$ and $h_2[n]=\delta[n]+\delta[n-1]$ are connected in cascade, where $\delta[n]$ is the Kronecker delta. The impulse response of the cascaded system is

(A) $\delta[n]\delta[n-1]+\delta[n-2]\delta[n+1]$ (B) $\delta[n-2]+\delta[n+1]$

(C) $\delta[n-2]+\delta[n-1]+\delta[n]+\delta[n+1]$ (D) $\delta[n-1]\delta[n]+\delta[n+1]\delta[n-1]$

Key: **(C)**

Sol: It is given that

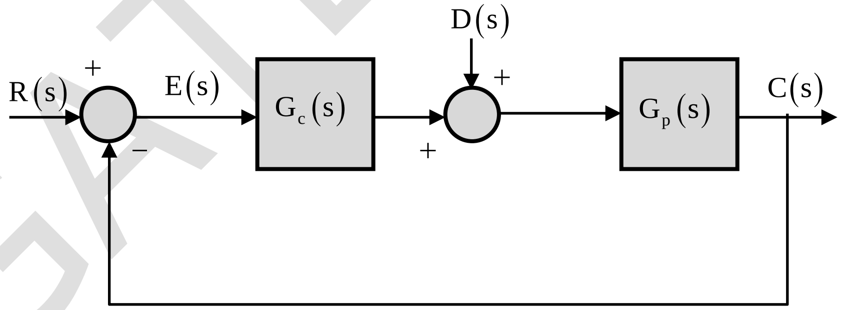
$$h_1(n)=\delta(n-1)+\delta(n+1)$$
$$h_2(n)=\delta(n)+\delta(n-1)$$

Then $h(n)=h_1(n)*h_2(n)$

$$=\{\delta(n-1)*[\delta(n)+\delta(n-1)]\}+\{\delta(n+1)*[\delta(n)+\delta(n-1)]\}$$
$$=[\delta(n-1)*\delta(n)]+[\delta(n-1)*\delta(n-1)]+[\delta(n+1)*\delta(n)]+[\delta(n+1)*\delta(n-1)]$$
$$=\delta(n-1)+\delta(n-2)+\delta(n+1)+\delta(n) \quad [\because \delta(n-n_o)*\delta(n-n_1)=\delta[n-(n_0+n_1)]]$$
$$=\delta(n-2)+\delta(n-1)+\delta(n)+\delta(n+1)$$

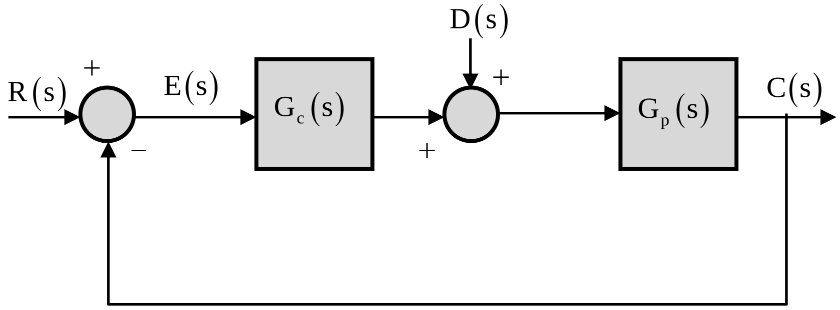
46. In the given figure, plant $G_p(s)=\dfrac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}$ and compensator $G_c(s)=K\left(\dfrac{1+T_1s}{1+T_2s}\right)$.

The external disturbance input is D(s). It is desired that when the disturbance is a unit step, the steady-error should not exceed 0.1 unit. The minimum value of K is ___________. (Round off to 2 decimal places).



Key: **(-10.45)**

Sol: The given block diagram is



$$G_p(s)=\frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)},\ G_c(s)=k\left(\frac{1+T_1s}{1+T_2s}\right)$$

When D(s) is unit step the steady state error should be at maximum 0.1, we need to obtain minimum value of k.

Since there are 2 inputs (R(s), D(s)), we want E(s) due to D(s), so R(s) should be made 0

$$\frac{E(s)}{D(s)}=\frac{G_p(s)}{1+G_pG_c(s)}$$

$$E(s)=\frac{-G_p(s)}{s\left(1+G_p(s)G_0(s)\right)}\qquad\left(\because D(s)=\frac{1}{s}\right)$$

Steady sate error $e(\infty)=\lim\limits_{s\to 0}sE(s)=\lim\limits_{s\to 0}\frac{-G_p(s)}{1+G_p(s)G_c(s)}$

$$\Rightarrow e(\infty)=\lim_{s\to 0}\frac{\frac{-2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}}{1+k\left(\frac{1+T_1s}{1+T_2s}\right)\frac{2.2}{(1+0.1s)(1+0.4s1+1.2s)}}=\frac{-2.2}{1+2.2k}$$

We need $e(\infty)\le 0.1$

$$\frac{-2.2}{1+2.2k}\le 0.1$$
$$\Rightarrow -2.2\le 0.1+0.22k$$
$$\Rightarrow -2.3\le 0.22k$$

$$0.22k\ge -2.3$$
$$k\ge\frac{-2.3}{0.22}$$
$$k\ge -10.45$$

Minimum value of k is –10.45.

47. In the open interval (0, 1), the polynomial $p(x) = x^4 - 4x^3 + 2$ has

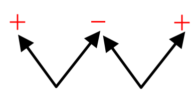
(A) two real roots (B) three real roots

(C) one real root (D) no real roots

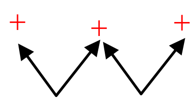
Key: (A)

Sol: $p(0) = z > 0$ and $p(1) = 1 - 4 + 2 = -1 < 0$

$\therefore$ one of the roots lies between 0 and 1

Signs of $p(x)$: 

$\Rightarrow$ two sign changes in $p(x)$

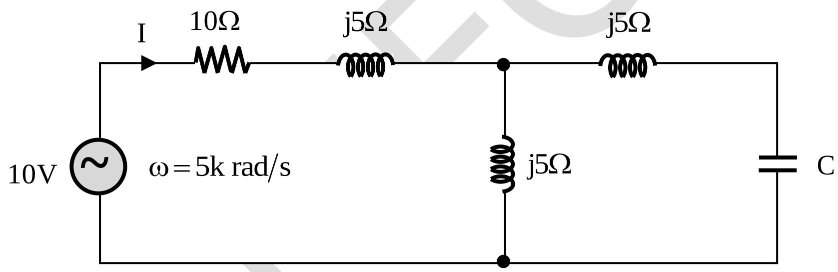
Signs of $p(-x)$: 

$\Rightarrow$ no sign changes in $p(-x)$

$\therefore$ P(x) has at most two positive roots and no negative root. We know that complex roots occur in pairs and p(x) is a 4^{th} degree polynomial.

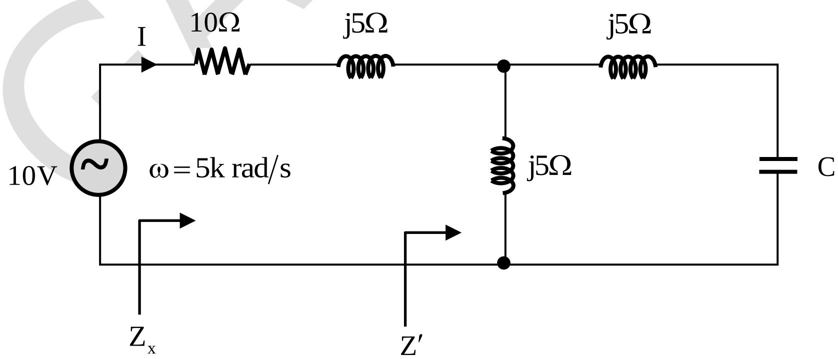
Hence p(x) has two real roots.

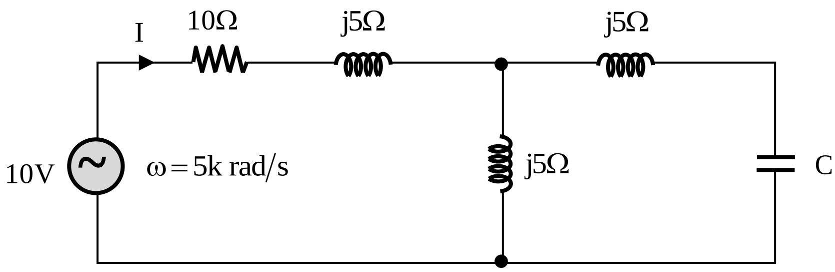
48. In the given circuit, the value of capacitor C that makes current I = 0 is ______ μF.



Key: (20)

Sol: In the following network we need to obtain value of C for which I = 0





If $Z' = \infty$ then $Z_x = \infty$ and hence $I = \frac{V}{Z_x} = 0A$

$$Z' = j5 \,||\, \left(j5 + \frac{1}{j\omega C}\right) = \frac{j5\left(j5 + \frac{1}{j\omega C}\right)}{j5 + j5 + \frac{1}{j\omega C}}$$

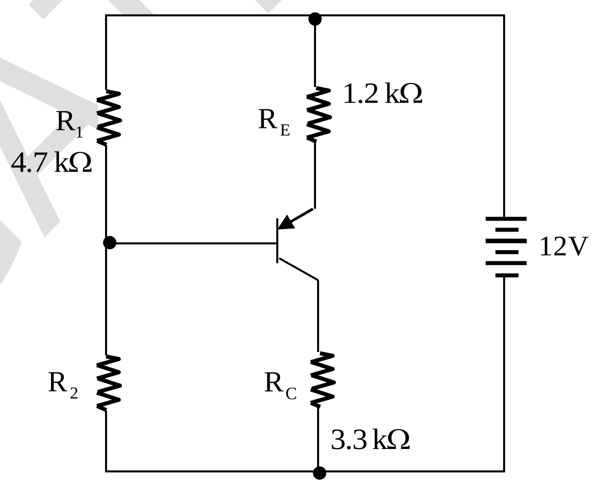
For $Z' = \infty$ we need $j5 + j5 + \frac{1}{j\omega C} = 0$

$$\Rightarrow j10 = \frac{+j}{\omega C}$$

$$\Rightarrow C = \frac{1}{10\omega} = \frac{1}{10 \times 5 \times 10^3} = 20\,\mu F$$

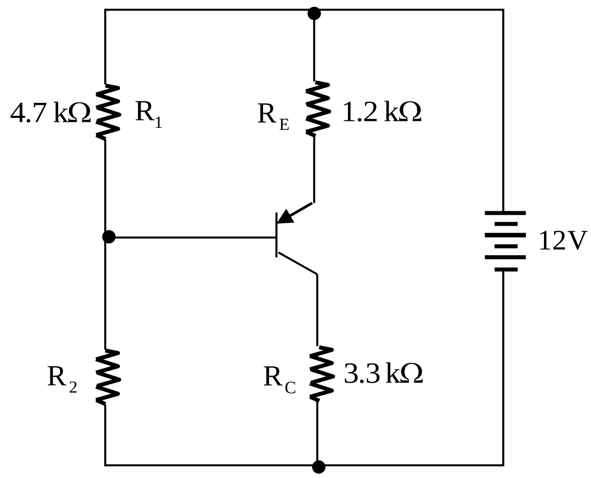
For current I = 0 we need $C = 20\,\mu F$.

49. In the BJT circuit shown, beta of the PNP transistor is 100. Assume $V_{BE} = -0.7V$. The voltage across R_C will be 5V when R_2 is ___________ kΩ. (Round of to 2 decimal places).



Key: (17.53)

Sol:



$$\text{Given } V_C = 5V, \text{ So, } I_C = \frac{V_C}{R_C} = \frac{5}{3.3} mA$$

$$I_E = \frac{I_C}{\alpha} = \left(\frac{\beta+1}{\beta}\right) I_C = \frac{101}{100} \times \frac{5}{3.3} mA = 1.53\, mA$$

$$V_E = 12V - 1.53 \times 1.2$$
$$V_E = 10.164V$$

$$\text{Given, } V_{BE} = -0.7V$$
$$V_B - V_E = -0.7V$$

$$\Rightarrow V_B = V_E - 0.7V = 10.164V - 0.7V = 9.464V$$

By applying Thevenin's equivalent at the input bias network.

$$V_B = V_{Th} = \frac{R_2}{R_1 + R_2} \times V_{CC}$$

$$9.464V = \frac{R_2}{4.7k + R_2} \times 12V$$

$$\frac{4.7k + R_2}{R_2} = \frac{12}{9.464}$$

$$\left[1 + \frac{4.7k}{R_2}\right] = \frac{12}{9.464}$$

$$\frac{4.7k}{R_2} = \frac{12}{9.464} - 1 = 0.267$$

$$\boxed{\therefore R_2 = 17.53k\Omega}$$

50. Suppose the circle $x^3+y^2=1$ and $(x-1)^2+(y-1)^2=r^2$ intersect each other orthogonally at the point (u, v). The u + v = _________,

Key: **(1)**

Sol: Let $f(x,y)=x^2+y^2=1......(1)$ and $g(x,y)=(x-1)^2+(y-1)^2=r^2$ ……(2)

Then $\frac{dy}{dx}=\frac{-\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}=\frac{-2x}{2y}=-\frac{x}{y}$ is the slope of curve (1)

And $\frac{dy}{dx}=\frac{-\frac{\partial g}{\partial x}}{\frac{\partial g}{\partial y}}=\frac{-2(x-1)}{2(y-1)}=\frac{-(x-1)}{y-1}$ is the slope of curve (2)

For orthogonal curves, product of slopes is –1

$\Rightarrow \frac{x}{y}\times\frac{(x-1)}{(y-1)}=-1$

$\Rightarrow x^2-x=-y^2+y$

$\Rightarrow x^2+y^2=x+y$

$\Rightarrow x+y=1 \quad (\because x^2+y^2=1)$

Since (1) and (2) intersect at (u, v)

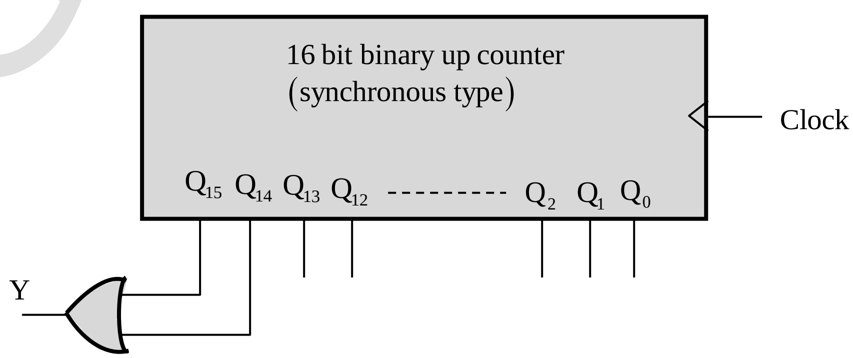
$\therefore$ u + v = 1

51. A 16-bit synchronous binary up-counter is clocked with a frequency f_{CLK}. The two most significant bits are OR-ed together to form an output Y. Measurement show that Y is periodic, and the duration for which Y remains high in each period is 24ms. The clock frequency f_{CLK} is ______ MHz.

(Round off to 2 decimal places).

Key: **(2.048)**

Sol: From the given information we can draw the following block diagram



It is given that Y = 1 for 24 m.sec in every period of Y. We need to obtain the clock frequency.

Since it's a 16-bit synchronous binary up counter, when it will receive clocks it will change its states from [00…000 to 11…111] i.e., total 2^{16} number of distinct states in one counting cycle.

Y will be 0 only when Q_{15} and Q_{14} both are zero simultaneously else Y will be 1.

So out of 2^{16} number of states, the number of states when $Q_{15} = Q_{14} = 0$ is 2^{14} (i.e., $Q_{15} = Q_{14} = 0$, but rest of the 14 bits Q_{13} to Q_0 can vary minimum to maximum).

Total 2^{16} number of states

Out of 2^{16} states in 2^{14} states Y= 0

Out of 2^{16} states $\left(2^{16} - 2^{14}\right)$ states Y = 1

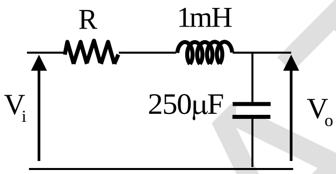
$$\left(2^{16} - 2^{14}\right)T_{clk} = 24\,\text{m.sec}$$

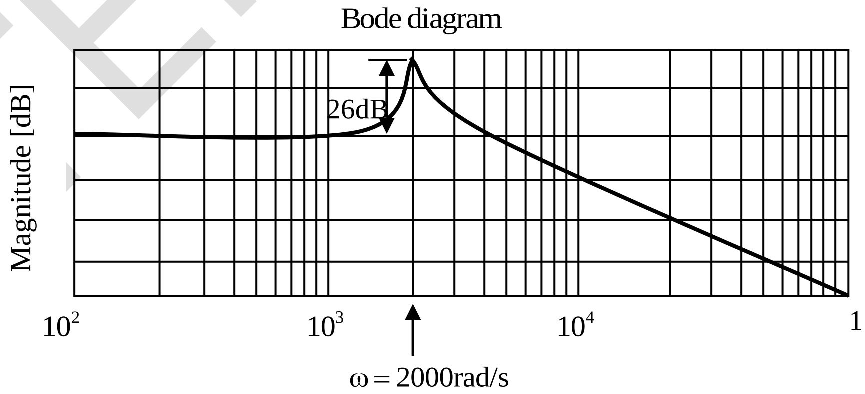
$$T_{clk} = \frac{24 \times 10^{-3}}{2^{16} - 2^{14}} = 488 \times 10^{-9}\,\text{sec} = 488\,\text{n.sec}$$

$$f_{clk} = \frac{1}{T_{clk}} = \frac{1}{488 \times 10^{-9}} = 2.048\,\text{MHz}$$

52. The bode magnitude plot for the transfer function $\frac{V_o(s)}{V_i(s)}$ of the circuit is as shown.

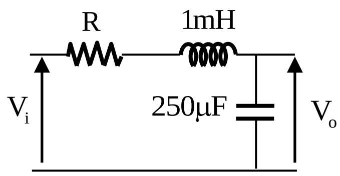
The value of R is ____________Ω. (Round off to 2 decimal places).





Key: **(0.1)**

Sol: For the following RLC circuit, the gain at frequency 2000rad/sec is given 26dB. We need to obtain value of R



When the operating frequency is 2000 rad/sec

$$Z_L = j\omega L = j\times 2000\times 1\times 10^{-3} = j2$$

$$Z_c = \frac{-j}{\omega C} = \frac{-1}{200\times 250\times 10^{-6}} = -j2$$

In general

$$V_o(j\omega) = \frac{Z_C}{R+Z_L+Z_C}V_i(j\omega)$$

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-j2}{R+j2-j2}\left(\text{at } \omega = 2000\ \text{r/sec}\right)$$

$$\Rightarrow \left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \frac{2}{R} \qquad ...(1)$$

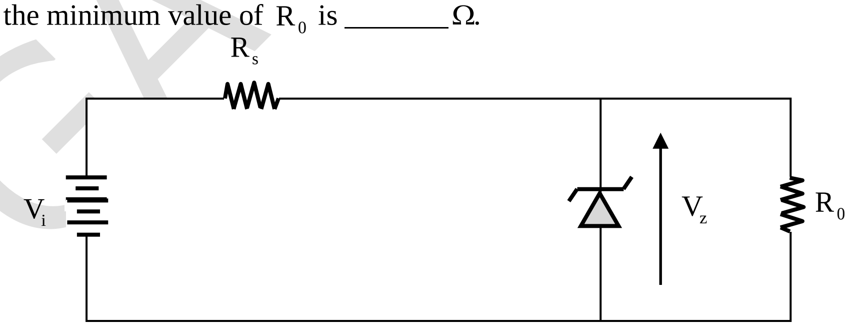
It is given that $20\log\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = 26$

$$\Rightarrow \left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = 10^{\frac{26}{20}} \qquad \left(\text{at } \omega = 2000\ \text{rad/sec}\right)$$

$$\Rightarrow \left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = 19.95 \qquad ...(2)$$

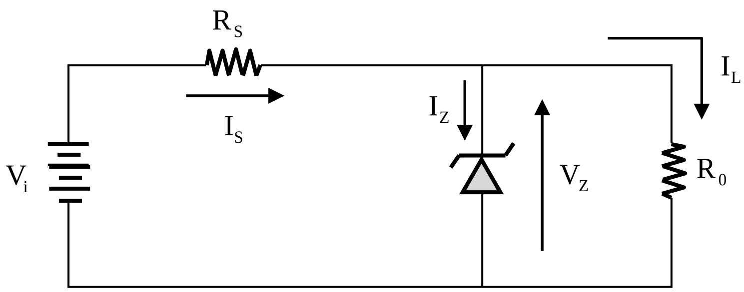
Equating equation (1) and (2) $\frac{2}{R} = 19.95 \Rightarrow R = \frac{2}{19.95} \Rightarrow R = 0.1\Omega$

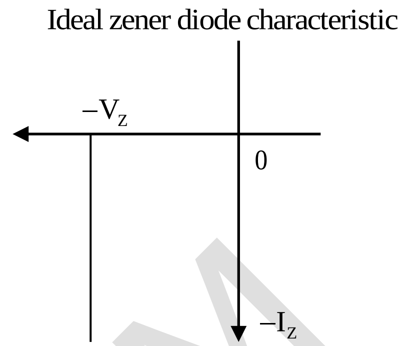
53. In the circuit shown, a 5V Zener diode is used to regulate the voltage across load R_0. The input is an unregulated DC voltage with a minimum value of 6V and a maximum value of 8V. The value of R_S is $6\,\Omega$. . The zener diode has a maximum rated power dissipation of 2.5 W. Assuming the zener diode to be ideal, the minimum value of R_0 is _______Ω.



Key: (10)

Sol:





For ideal Zener diode $I_{z\min} = I_{knee} = 0mA$

Given, $P_{Z\max} = I_{Z\max} V_Z$

$$I_{z\max} = \frac{P_{Z\max}}{V_Z} = \frac{2.5}{5} = 0.5A$$

$$R_{0(\min)} = \frac{V_Z}{I_{L(\max)}} = ?$$

$$I_S = I_Z + I_L$$

$$I_{S(\max)} = I_{Z\min} + I_{L(\max)}$$

$$\frac{V_{i(\max)} - V_Z}{R_S} = 0 + I_{L(\max)}$$

$$\therefore I_{L(\max)} = \frac{8-5}{6} = \frac{3}{6} = 0.5A$$

$$R_{0(\min)} = \frac{5}{0.5} = 10\Omega$$

$$\boxed{\therefore R_{0(\min)} = 10\Omega}$$

54. Let $(-1-j),(3-j),(3+j)$ and $(-1+j)$ be the vertices of a rectangle C in the complex plane. Assuming that C is traversed in counter-clockwise direction, the value of the contour integral $\oint_C \frac{dz}{z^2(z-4)}$ is

(A) 0 (B) $\frac{-j\pi}{8}$ (C) $\frac{j\pi}{16}$ (D) $\frac{j\pi}{2}$

Key: **(B)**

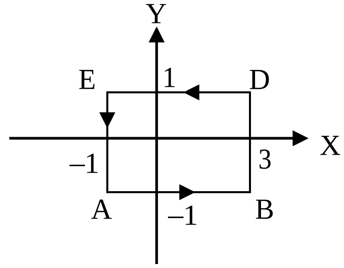
Sol: Let $A(-1,-1) = -1-j; B(3,-1); D(3,1)$ and $E(-1,1)$ be the vertices of rectangle

$$z^2(z-4) = 0 \Rightarrow z = 0, 4$$

$$\therefore \oint_C \frac{dz}{z^2(z-4)} = \oint_C \frac{\left(\frac{1}{z-4}\right)}{(z-0)^2} dz$$

$$= \frac{2\pi i}{1!}\left(\frac{d}{dz}\left(\frac{1}{z-4}\right)\right)_{\text{at } z=0} \quad (\text{Using Cauchy's integral formula})$$

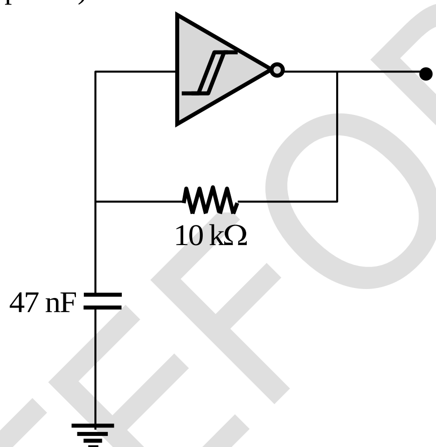
$$= 2\pi i x\left(\frac{-1}{(z-4)^2}\right)_{\text{at } z=0} = \frac{-\pi i}{8} \text{ or } \frac{-j\pi}{8}$$



Only $z = 0$ lies inside the curve

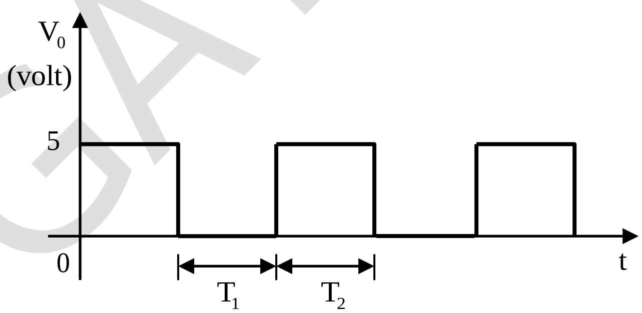
55. A CMOS Schmitt-trigger inverter has a low output level of 0V and a high output level of 5V. It has input thresholds of 1.6V and 2.4V. The input capacitance and output resistance of the Schmitt-trigger are negligible. The frequency of the oscillator shown is _________Hz.

(Round off to 2 decimal places).



Key: (3166.20)

Sol:



$$T = T_1 + T_2 = 1, \ f = \frac{1}{T}$$

For $T_1 : V_C(0^-) = 2.4V, \ V_C(\infty) = 0V$

$$V_C(t)=V_C(\infty)+\left(V_C(0^-)-V_C(\infty)\right)e^{\frac{-t}{RC}}$$

At $t=T_1,\ V_C=1.6V$

$$1.6V=0+(2.4-0)e^{\frac{-T_1}{RC}}$$

$$e^{\frac{-T_1}{RC}}=\frac{1.6}{2.4}$$

$$\frac{-T_1}{RC}=\ell n\left(\frac{1.6}{2.4}\right)=-\ell n\left(\frac{2.4}{1.6}\right)$$

$$\therefore\ T_1=RC\ell n(1.5)=10\times10^3\times47\times10^{-9}\times\ell n(1.5)=1.9\times10^{-4}\sec$$

For T_2 : $V_C(0^-)=1.6V,\ V_C(\infty)=5V$

$$V_C(t)=V_C(\infty)+\left(V_C(0^-)\right)-V_C(\infty)e^{-\frac{t}{RC}}$$

At $t=T_2;\ V_C=2.4V$

$$2.4V=5+(1.6-5)e^{\frac{-T_2}{RC}}$$

$$2.4-5=-3.4e^{\frac{-T_2}{RC}}$$

$$-2.6=-3.4e^{\frac{-T_2}{RC}}$$

$$e^{\frac{-T_2}{RC}}=\frac{2.6}{3.4}$$

$$\frac{-T_2}{RC}=\ell n\left(\frac{2.6}{3.4}\right)=-\ell n\left(\frac{3.4}{2.6}\right)$$

$$T_2=RC\ell n(1.307)$$

$$=10\times10^3\times47\times10^{-9}\times\ell n(1.307)$$

$$=1.258\times10^{-4}\sec$$

$$T=T_1+T_2=(1.9+1.258)\times10^{-4}\sec=3.151\times10^{-4}\sec$$

$$f=\frac{1}{T}=\frac{1}{3.151\times10^{-4}}Hz=3166.201\,Hz$$

$$\boxed{f=3166.20Hz}$$